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Distributions Table

Discrete Distributions

The geometric distribution counts the number of trials up to and including the first success. Its MGF is finite only for $\theta < -\ln(1-p)$.

Distribution	Support	$\mathrm{P}(X=k)$	PGF $G(z)$	MGF $M(\theta)$	Mean	Variance
Bernoulli(p)	$\{0,1\}$	$p^k(1-p)^{1-k}$	$1-p+pz$	$1-p+pe^{\theta}$	p	$p(1-p)$
Binomial(n,p)	$\{0,\ldots,n\}$	$\binom{n}{k}p^k(1-p)^{n-k}$	$(1-p+pz)^n$	$(1-p+pe^{\theta})^n$	np	$np(1-p)$
Geometric(p)	$\{1,2,\ldots\}$	$(1-p)^{k-1}p$	$\frac{pz}{1-(1-p)z}$	$\frac{pe^{\theta}}{1-(1-p)e^{\theta}}$	$\frac{1}{p}$	$\frac{1-p}{p^2}$
Poisson(λ)	$\{0,1,\ldots\}$	$e^{-\lambda}\frac{\lambda^k}{k!}$	$e^{\lambda(z-1)}$	$e^{\lambda(e^{\theta}-1)}$	λ	λ

Continuous Distributions

The MGFs of the exponential and gamma distributions are finite only for $\theta < \lambda$, and that of the chi-squared distribution only for $\theta < 1/2$. The lognormal and Student t distributions have no MGF: $\mathrm{E}[e^{\theta X}]$ is infinite for every $\theta > 0$.

Distribution	Support	Density $f(x)$	MGF $M(\theta)$	Mean	Variance
Uniform(a, b)	$[a, b]$	$\frac{1}{b-a}$	$\frac{e^{\theta b}-e^{\theta a}}{\theta(b-a)}$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$
Exp(λ)	$[0, \infty)$	$\lambda e^{-\lambda x}$	$\frac{\lambda}{\lambda-\theta}$	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$
Gamma(α, λ)	$(0, \infty)$	$\frac{\lambda^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\lambda x}$	$\left(\frac{\lambda}{\lambda-\theta}\right)^\alpha$	$\frac{\alpha}{\lambda}$	$\frac{\alpha}{\lambda^2}$
$\mathcal{N}(\mu, \sigma^2)$	$\mathbb{R}$	$\frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$	$e^{\mu\theta+\frac{1}{2}\sigma^2\theta^2}$	μ	σ^2
Chi-squared(k)	$(0, \infty)$	$\frac{(1/2)^{k/2}}{\Gamma(k/2)} x^{k/2-1} e^{-x/2}$	$(1-2\theta)^{-k/2}$	k	$2k$
Lognormal(μ, σ^2)	$(0, \infty)$	$\frac{1}{x\sqrt{2\pi\sigma^2}} e^{-\frac{(\ln x-\mu)^2}{2\sigma^2}}$		$e^{\mu+\frac{1}{2}\sigma^2}$	$e^{2\mu+\sigma^2}(e^{\sigma^2}-1)$
Student $t(\nu)$	$\mathbb{R}$	$\propto \left(1+\frac{x^2}{\nu}\right)^{-\frac{\nu+1}{2}}$		0 if $\nu > 1$	$\frac{\nu}{\nu-2}$ if $\nu > 2$

The characteristic function of each distribution with an MGF follows by setting $\theta = iu$. The cumulant generating function is $K(\theta) = \ln M(\theta)$, and for $X \sim \mathcal{N}(\mu, \sigma^2)$ the lognormal moments follow from $\mathrm{E}[e^{nX}] = M(n) = e^{n\mu+\frac{1}{2}n^2\sigma^2}$.