Utility Theory Under Uncertainty

Lorenzo Naranjo

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Introduction

These notes develop expected utility, the standard tool used throughout finance to model how an investor evaluates a risky outcome. We start with a few common utility functions and the notion of risk aversion, and use Jensen's inequality to show that risk aversion is equivalent to a strictly concave utility function. We then introduce the insurance premium and the certainty equivalent as ways to turn an agent's risk aversion into a dollar amount, and use a Taylor expansion to derive the coefficients of absolute and relative risk aversion, which measure risk aversion locally around a given wealth level. The notes close with the risk premium, the compensation an agent must be offered to accept a risky gamble, and relate it back to the insurance premium.

In a single period model, agents must decide today how much to consume and how much to save for later. In this note we take that decision as given, and assume that agents derive their utility from consumption at the end of the period. As is common in finance, consumption is represented by a single good. Agents prefer more to less, but the marginal utility of each additional unit of consumption is decreasing, i.e., the last bite is never as good as the first one.

We capture this intuition by a utility function $u : \mathbb{R}^+ \to \mathbb{R}$, where $\mathbb{R}^+ = [0, \infty)$. The domain of the utility function is real consumption and therefore cannot be negative. In the following, we assume that $u(c)$ is continuous and differentiable of all orders. The level of utility is not important, and can even be negative, but we assume that the utility function is increasing and strictly concave.[1] Mathematically, it must be the case that $u'(c) > 0$ and $u''(c) < 0$. In some cases it will also be useful to consider utility functions such that $u'(0) = \infty$, that is, in starvation an extra unit of consumption provides an infinite amount of extra utility.

[1] A function $f : D \to \mathbb{R}$ is concave if

$$f(tx_1 + (1-t)x_2) \geq tf(x_1) + (1-t)f(x_2)$$

for all $x_1, x_2 \in D$ and $0 \leq t \leq 1$. The function is strictly concave if the inequality is strict.

Recall the power/log utility function introduced in the Fisher Model note, including why log utility is the $\gamma \to 1$ limit of power utility rather than just a separate special case, along with the exponential utility function introduced there:

$$u(c) = \frac{c^{1-\gamma} - 1}{1-\gamma}, \quad \gamma > 0, \gamma \neq 1, \qquad u(c) = \ln(c), \qquad u(c) = -e^{-ac}.$$

The marginal utility of power utility is $u'(c) = c^{-\gamma}$. We will see that power/log and exponential utility induce different attitudes under uncertainty.

Expected Utility and Risk Aversion

The analysis in the next two sections closely follows Chapter 1 in Ingersoll (1987). Denote by $\tilde{W}$ the wealth of the agent at the end of the period. This wealth is generated by investing a certain amount today. In general, $\tilde{W}$ is unknown today and can be thought of as a random variable defined over a probability space (Ω, P). We use the tilde on top of W to emphasize that the wealth is unknown today. Since all wealth is consumed at the end of the single period, $\tilde{W}$ plays the role of end-of-period consumption, and we use $u(\tilde{W})$ interchangeably with $u(c)$ from here on.

When faced with uncertainty, the utility of final consumption is also random. Thus, the agent is not so much worried about the level of consumption she is going to get, but rather she is more worried about the utility she might get from that level of consumption. We can model the agent's behavior using the concept of **expected utility**. The utility derived by a random consumption $\tilde{W}$ is given by

$$U(\tilde{W}) = \mathrm{E}(u(\tilde{W})).$$

Thus, we have that $\tilde{W}_1 \gtrsim \tilde{W}_2$ if $U(\tilde{W}_1) \geq U(\tilde{W}_2)$, and $\tilde{W}_1 \sim \tilde{W}_2$ if $U(\tilde{W}_1) = U(\tilde{W}_2)$. Note that the expected utility of a certain level of wealth W is just $u(W)$.

Consider now a random variable $\tilde{\varepsilon}$ such that $\mathrm{E}(\tilde{\varepsilon}) = 0$ and $\mathrm{V}(\tilde{\varepsilon}) > 0$. We say that an agent is risk-averse if she prefers a certain level of wealth W over a random payoff with the same expected value. If we define $\tilde{W} = W + \tilde{\varepsilon}$, we have that $\mathrm{E}(\tilde{W}) = W$, but $\mathrm{V}(\tilde{W}) > 0 = \mathrm{V}(W)$.

Thus, the agent is risk-averse if

$$u(W) > \mathrm{E}(u(W + \tilde{\varepsilon})). \tag{1}$$

Figure 1 illustrates why this holds for a strictly concave utility function. Take a gamble that pays off $W - \Delta$ or $W + \Delta$ with equal probability, so that $\mathrm{E}(\tilde{W}) = W$. The chord connecting $(W - \Delta, u(W - \Delta))$ and $(W + \Delta, u(W + \Delta))$ lies entirely below the curve, and its midpoint gives $\mathrm{E}(u(\tilde{W}))$. Because the curve is concave, that midpoint sits below $u(W)$: the sure thing beats the gamble.

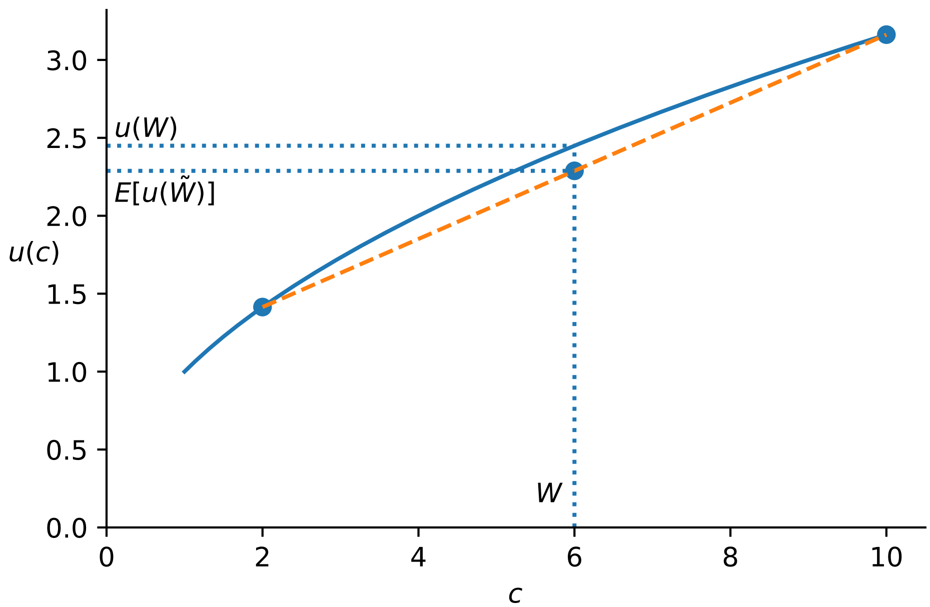

Figure 1: A concave utility function and Jensen's inequality.

In order to understand better the notion of risk-aversion, we will use the following result.

Property 1 (Jensen's Inequality). *Let $f : D \to \mathbb{R}$ be a twice-continuously differentiable and strictly concave function, and X a random variable defined in a probability space (Ω, P) such* *that the range of X is contained in the domain of f, and* $\mathsf{V}(X) > 0$. *Then we have that*[2]

$$f(\mathsf{E}(X)) > \mathsf{E}(f(X)).$$

Jensen's inequality shows that strict concavity in u implies risk-aversion. The converse is also true: risk-aversion implies that u must be strictly concave.[3]

Property 2 (Risk Aversion and Concavity of the Utility Function)**.** *An agent is risk-averse as defined in (1) if and only if her utility function is strictly concave.*

This equivalence is what makes strict concavity the defining mathematical feature of risk-averse preferences, and it opens the door to comparing how risk-averse different agents are, not just whether they are. The natural way to make that comparison concrete is to ask how much an agent would pay to get rid of a given risk altogether, which is the idea we turn to next.

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[2]Let $m = \mathsf{E}(X)$. Since f is strictly concave and differentiable, the tangent line at m lies strictly above the graph of f for all $x \neq m$:

$$f(x) < f(m) + f'(m)(x - m).$$

Setting $x = X$ and taking expectations,

$$\mathsf{E}(f(X)) < f(m) + f'(m)\ \mathsf{E}(X - m) = f(m) = f(\mathsf{E}(X)),$$

where $\mathsf{E}(X - m) = 0$ follows from $m = \mathsf{E}(X)$, and the inequality is strict because $\mathsf{V}(X) > 0$ ensures $X \neq m$ with positive probability.

[3]Consider a risk-averse investor with utility function u and a gamble $\tilde{\varepsilon}$ that pays $(1 - q)a$ with probability q and $-qa$ with probability $1 - q$. The gamble is fair since

$$\mathsf{E}(\tilde{\varepsilon}) = q(1 - q)a - (1 - q)qa = 0.$$

A risk-averse investor, though, dislikes the gamble, so

$$u(W) > \mathsf{E}(u(W + \tilde{\varepsilon})) = q\, u(W + (1 - q)a) + (1 - q)\, u(W - qa).$$

Because $W = q(W + (1 - q)a) + (1 - q)(W - qa)$ for all values of a and q such that $W + (1 - q)a$ and $W - qa$ are in the domain of u, this says that u evaluated at a convex combination of two points is larger than the corresponding convex combination of u-values, i.e., u is strictly concave. This specific family of gambles is enough to cover the general case: for any x_1, x_2 in the domain of u and any $t \in (0, 1)$, setting $q = t$, $W = tx_1 + (1 - t)x_2$, and $a = x_1 - x_2$ gives $W + (1 - q)a = x_1$ and $W - qa = x_2$, so the inequality above becomes $u(tx_1 + (1 - t)x_2) > tu(x_1) + (1 - t)u(x_2)$.

The Insurance Premium

In many situations, we are forced to take on a risky gamble. For example, if you buy a car you face the risk of an accident that can result in costly repairs. Many people choose to pay for insurance and thereby reduce that risk. In our framework, we define the insurance premium as the maximum amount that an agent is willing to pay to eliminate the risk.

To formalize this notion, consider an asset with value W. After you buy the asset, you can either face the risk of owning the asset producing a wealth of $W + \tilde{\varepsilon}$, or pay Π_i to insure the asset, which guarantees a certain wealth of $W - \Pi_i$. An agent is indifferent between the two choices if

$$u(W - \Pi_i) = \mathsf{E}(u(W + \tilde{\varepsilon})). \tag{2}$$

The value of Π_i that solves (2) is called the **insurance premium**. When comparing two agents' attitudes towards risk, the agent who is willing to pay the most for insurance is **more risk averse** than the other.

In the previous analysis, since the agent is indifferent between the risky gamble and getting $W - \Pi_i$, we call this certain amount, $W - \Pi_i$, the **certainty equivalent**.

Example 1. Consider an investor with logarithmic utility $u(W) = \ln W$ and current wealth $W = \$10{,}000$. There is a 10% probability that a loss event reduces wealth to $\$1{,}000$, and a 90% probability that wealth remains at $\$10{,}000$. Two insurance policies are available:

- **Plan A**: pays $\$5{,}000$ if the loss event occurs and costs $\$500$ upfront.
- **Plan B**: pays $\$8{,}000$ if the loss event occurs and costs $\$800$ upfront.

Should the investor buy Plan A, Plan B, or no insurance?

Note first that both policies are actuarially fair: the premium equals the expected payout in each case ($0.1 \times \$5{,}000 = \$500$ and $0.1 \times \$8{,}000 = \$800$), so all three alternatives leave expected wealth unchanged at

$$\mathsf{E}(\widetilde{W}) = 0.9 \times \$10{,}000 + 0.1 \times \$1{,}000 = \$9{,}100.$$

Since expected wealth is the same in every case, a risk-averse investor's ranking depends only on how much dispersion remains across states, which by Jensen's inequality she wants to minimize.

Without insurance, wealth is $\$1{,}000$ in the bad state and $\$10{,}000$ in the good state, so

$$\mathrm{E}(u) = 0.1 \ln(1{,}000) + 0.9 \ln(10{,}000) = 8.9801.$$

Under Plan A, wealth net of the premium is $\$10{,}000 - \$500 = \$9{,}500$ in the good state and $\$1{,}000 - \$500 + \$5{,}000 = \$5{,}500$ in the bad state, so

$$\mathrm{E}(u) = 0.1 \ln(5{,}500) + 0.9 \ln(9{,}500) = 9.1044.$$

Under Plan B, wealth is $\$10{,}000 - \$800 = \$9{,}200$ in the good state and $\$1{,}000 - \$800 + \$8{,}000 = \$8{,}200$ in the bad state, so

$$\mathrm{E}(u) = 0.1 \ln(8{,}200) + 0.9 \ln(9{,}200) = 9.1155.$$

Translating each expected utility into a certainty equivalent, $CE = \exp(\mathrm{E}(u))$, gives

$$CE_{\text{none}} = \$7{,}943, \qquad CE_A = \$8{,}994, \qquad CE_B = \$9{,}095.$$

Since $CE_B > CE_A > CE_{\text{none}}$, the investor should choose **Plan B**. Intuitively, all three alternatives are fairly priced and share the same expected wealth, so the only thing that differs is how much each one compresses the gap between good- and bad-state wealth: no insurance leaves a $\$9{,}000$ gap, Plan A leaves a $\$4{,}000$ gap, and Plan B leaves only a $\$1{,}000$ gap. Because u is strictly concave, a risk-averse investor always prefers the fairly-priced contract that reduces this spread the most, which here is Plan B; full insurance, eliminating the gap entirely, would be even better if offered at the same fair price. □

Example 2. Suppose the economy will be in one of two states at the end of the period: a **boom**, or a **recession**, each equally likely. A risky asset would be worth $50 in the boom and $10 in the recession, and it currently trades at $30. Two investors, A and B, are each sizing up whether it's worth buying. To keep the example simple, assume this asset is each investor's only source of wealth at the end of the period, so that end-of-period wealth W coincides with the asset's payoff and we can evaluate $u(W)$ directly at $10 and $50.

Investor A has the utility function

$$u(W) = 10\ln(W),$$

while investor B has the linear utility function

$$u(W) = 2W + 5,$$

which makes B risk-neutral rather than risk-averse — a useful foil for A.

How much would each be willing to pay? The answer is each investor's certainty equivalent: the highest price that leaves them no worse off, in expected-utility terms, than passing on the asset.

For investor A, the expected utility of holding the asset is

$$E(U) = 0.5(10\ln(10)) + 0.5(10\ln(50)) = 31.0730.$$

Setting this equal to the utility of a sure payoff and solving pins down A's certainty equivalent:

$$\begin{aligned} 10\ln(CE) &= 31.0730 \\ \ln(CE) &= 3.1073 \\ CE &= \exp(3.1073) = \$22.36. \end{aligned}$$

So A values the asset at only $22.36 — well short of its $30 market price. Even though the asset pays $30 on average, the prospect of ending up with just $10 in a recession weighs heavily enough that A would rather not hold it.

Investor B sees things differently. Her expected utility is

$$E(U) = 0.5(2 \times 10 + 5) + 0.5(2 \times 50 + 5) = 65,$$

and solving

$$\begin{aligned} 2 \times CE + 5 &= 65 \\ 2 \times CE &= 60 \\ CE &= \$30 \end{aligned}$$

gives $CE = \$30$ — B is exactly indifferent at the market price. That's no coincidence: with linear utility, B cares only about expected payoffs, so her certainty equivalent must equal the asset's expected value, $0.5 \times 10 + 0.5 \times 50 = \$30$.

The \$7.64 gap between the two certainty equivalents ($30 - 22.36$) is the price of risk itself — the insurance premium A would pay to shed this gamble, which vanishes entirely for the risk-neutral B. □

Local Risk Aversion

Intuitively, a function that is more concave should induce more risk aversion than a function that is less concave. We can formalize this intuition by looking at the insurance premium for a gamble with a very small variance.

Let $\mathrm{E}(\tilde{\varepsilon}) = 0$ and $\mathrm{V}(\tilde{\varepsilon}) > 0$. We know that the insurance premium Π_i solves $u(W - \Pi_i) = \mathrm{E}(u(W + \tilde{\varepsilon}))$.

First, do a Taylor expansion of first order of $u(W - \Pi_i)$ around W:

$$\begin{aligned} u(W - \Pi_i) &\approx u(W) + u'(W)(W - \Pi_i - W) \\ &= u(W) - u'(W)\Pi_i. \end{aligned} \tag{3}$$

Second, do a Taylor expansion of second order of $u(W + \tilde{\varepsilon})$ around W:

$$\begin{aligned} u(W + \tilde{\varepsilon}) &\approx u(W) + u'(W)(W + \tilde{\varepsilon} - W) + \frac{1}{2}u''(W)(W + \tilde{\varepsilon} - W)^2 \\ &= u(W) + u'(W)\tilde{\varepsilon} + \frac{1}{2}u''(W)\tilde{\varepsilon}^2 \\ \mathrm{E}(u(W + \tilde{\varepsilon})) &\approx u(W) + \frac{1}{2}u''(W)\sigma_{\varepsilon}^2. \end{aligned} \tag{4}$$

Equating (3) and (4) we find that:

$$\Pi_i \approx -\frac{1}{2}\frac{u''(W)}{u'(W)}\sigma_\varepsilon^2. \tag{5}$$

The previous expression shows that for an initial wealth of W, the insurance premium depends positively on the local curvature of the utility function at that point as measured by $-u''(W)$.

We denote by

$$ARA = -\frac{u''(W)}{u'(W)}$$

the coefficient of absolute risk-aversion, and by

$$RRA = -\frac{u''(W)}{u'(W)}W$$

the coefficient of relative risk-aversion.

Equation (5) can be rewritten as

$$\Pi_i \approx \frac{1}{2}ARA \cdot \sigma_\varepsilon^2,$$

so ARA is literally the dollar insurance premium the agent is willing to pay per unit of variance: an agent with a higher ARA pays more, in absolute dollar terms, to eliminate any given gamble. Because ARA can itself depend on W, it is natural to ask how risk-aversion changes with wealth. An agent with decreasing absolute risk-aversion (DARA) requires a smaller dollar premium as wealth grows, since risky positions become less burdensome as the agent becomes wealthier — the pattern favored by most empirical evidence and introspection.

The factor of W in RRA has a similar interpretation once we look at proportional rather than dollar gambles. Consider a proportional gamble $\tilde{\varepsilon} = W\tilde{\delta}$, where $\tilde{\delta}$ is a percentage shock with $\mathrm{E}(\tilde{\delta}) = 0$ and $\mathrm{V}(\tilde{\delta}) = \sigma_\delta^2$. Substituting into (5) gives

$$\Pi_i \approx \frac{1}{2}ARA \cdot W^2\sigma_\delta^2 = \frac{1}{2}RRA \cdot W\sigma_\delta^2,$$

and dividing by W gives the fraction of wealth the agent would sacrifice to eliminate the proportional gamble:

$$\frac{\Pi_i}{W} \approx \frac{1}{2}RRA \cdot \sigma_\delta^2.$$

RRA therefore measures the premium per unit variance for proportional gambles, expressed as a share of wealth. This is also why an agent with constant relative risk-aversion (CRRA) maintains the same fraction of wealth invested in risky assets regardless of her wealth level, a portfolio-choice result we state here for intuition and derive formally in a later note.

Example 3. Take

$$u(W) = \frac{W^{1-\gamma} - 1}{1-\gamma}$$

Then $u'(W) = W^{-\gamma}$ and $u''(W) = -\gamma W^{-\gamma-1}$, implying that

$$RRA = -\left(\frac{-\gamma W^{-\gamma-1}}{W^{-\gamma}}\right) W = \gamma$$

Power utility is an example of a function that exhibits constant relative risk-aversion. □

Example 4. For exponential utility $u(W) = -e^{-aW}$ with $a > 0$, we have $u'(W) = ae^{-aW}$ and $u''(W) = -a^2 e^{-aW}$, so

$$ARA = \frac{a^2 e^{-aW}}{ae^{-aW}} = a.$$

The coefficient of absolute risk-aversion is constant (equal to a) and independent of W. Exponential utility therefore exhibits **constant absolute risk-aversion** (CARA). A direct implication is that the insurance premium $\Pi_i \approx \frac{1}{2} a\sigma_\varepsilon^2$ does not depend on the agent's wealth: a billionaire and a middle-class worker with the same CARA parameter a would pay identical dollar amounts to insure against the same gamble. This is analytically convenient but empirically implausible, since most people become less risk-averse in absolute terms as they accumulate wealth. □

Figure 2 puts these two examples side by side. The power-utility agent's ARA falls as wealth grows (DARA), while the exponential-utility agent's ARA stays flat at a no matter how wealthy she becomes (CARA). At low wealth the CRRA agent can be far more risk-averse than the CARA agent; at high wealth the ordering flips, since the CRRA agent's ARA keeps shrinking while the CARA agent's does not.

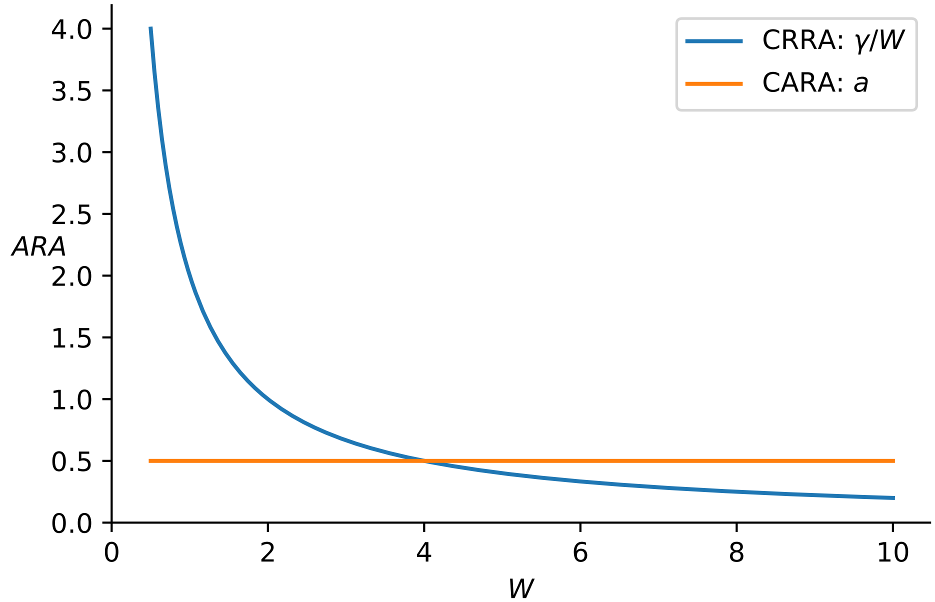

Figure 2: Absolute risk-aversion for CRRA (power) versus CARA (exponential) utility.

The Risk Premium

The insurance premium Π_i answers one specific question: how much would an agent pay to shed a fair gamble $\tilde{\varepsilon}$ she is already exposed to? A closely related but distinct question arises when the agent does *not* currently bear the gamble and must instead be persuaded to accept it: how much extra compensation would she need to be offered on top of the gamble before she is willing to take it on? The answer is the **compensatory risk premium** Π_c, defined by

$$u(W) = \mathsf{E}\left(u(W + \Pi_c + \tilde{\varepsilon})\right). \tag{6}$$

Here Π_c is the amount that, added to the fair gamble, exactly offsets the agent's distaste for its risk, leaving her indifferent between staying at the certain wealth W and accepting the sweetened gamble $W + \Pi_c + \tilde{\varepsilon}$.

Although Π_i and Π_c both measure aversion to the same gamble $\tilde{\varepsilon}$ at the same wealth W, they are generally *not* equal, because they price risk from opposite sides: Π_i is what the agent pays to get rid of a risk she holds, while Π_c is what she must be paid to take one on. The insurance premium is the natural quantity for casualty or liability insurance; the compensatory premium is the natural one in finance, since it corresponds to the extra expected return an investor must be offered to hold a risky asset instead of a riskless one.

For a small gamble, however, the two nearly coincide. Applying the same second-order Taylor argument used above for Π_i to (6) gives

$$\Pi_c \approx \frac{1}{2}\left(-\frac{u''(W)}{u'(W)}\right)\mathsf{V}(\tilde{\varepsilon}) = \frac{1}{2}\mathit{ARA}\cdot\mathsf{V}(\tilde{\varepsilon}),$$

which is exactly the local approximation for Π_i derived above in (5). So while Π_i and Π_c are conceptually distinct, for small risks they are approximately equal and both are governed by the same local curvature of u at W.

We can connect this back to Example 2 by taking the asset's expected payoff, $\$30$, as the reference wealth W and its $\pm\$20$ deviations as the fair gamble $\tilde{\varepsilon}$. Investor A's willingness to pay only $\$22.36$ for the asset is precisely the statement that she would demand an insurance premium of $\Pi_i = 30 - 22.36 = \$7.64$ to shed this risk, while the risk-neutral investor B has $\Pi_i = \$0$.

Together, Π_i, Π_c, and their shared local approximation via *ARA* give us a complete toolkit for turning an agent's risk aversion into a concrete price — whether that price is what she would pay to shed a risk, what she would need to be paid to accept one, or the expected return she would demand for holding a risky asset.

Practice Problems

These problems give you a chance to practice the concepts introduced in this chapter. Try to solve each one on your own before expanding the solution.

Solutions to all problems can be found at https://lorenzonaranjo.com/class-materials/investments/.

Problem 1 (The Insurance Decision Across Wealth Levels)**.** You have a logarithmic utility function $u(W) = \ln W$, and your current level of wealth is $\$5{,}000$.

a. Imagine you are in a situation where there's a 50/50 chance of either winning or losing $\$1{,}000$. You have the option to purchase insurance for $\$125$ that would entirely eliminate this risk. Would you choose to buy the insurance or take the gamble?
b. Let's say you chose to take the gamble in part (a) and ended up losing, which reduces your wealth to $\$4{,}000$. Now, if you are presented with the same gamble and the same insurance offer of $\$125$, would you opt to buy the insurance this time?

Problem 2 (Maximum Willingness to Pay for Insurance). An entrepreneur faces the following risks: a 10% chance that a fire will reduce her net worth to $\$1$, a 10% chance that a fire will lower it to $\$50{,}000$, and an 80% chance that nothing will happen, maintaining the business's value at $\$100{,}000$. Her utility function is logarithmic, given by $u(W) = \ln W$. She is considering an insurance policy that would provide a payout of $\$99{,}999$ in the first scenario, $\$50{,}000$ in the second, and nothing in the third. What is the maximum amount she would be willing to pay for this insurance policy?

Problem 3 (A Fair Lottery and Concavity). You are offered the possibility to participate at the following lottery:

Gain	Probability
2	0.5
0	0.5

The cost of participating at the lottery is 1 unit of consumption. If you choose not to participate, you keep your unit of consumption.

a. Is this a fair gamble?
b. Show that the decision not to participate at the gamble implies that your utility function is concave.

Problem 4 (Mean-Preserving Spreads and Risk Aversion). Consider gamble A:

Gain	Probability
-2	0.09
4	0.30
10	0.40
16	0.21

From A, we can construct another gamble B by adding white noise to a number of outcomes. Indeed, we can replace the outcome 4 by the gamble A':

- 3 with probability 1/2
- 5 with probability 1/2

with E$(A') = 4$. In the same manner, we can replace outcome 16 with gamble A'':

- 12 with probability 1/3
- 18 with probability 2/3

with E$(A'') = 16$.

Show formally why any risk averse individual prefers gamble A to gamble B.

Problem 5 (Divergent Beliefs and the Incentive to Litigate). A distant relative in Europe has recently passed away, leaving behind an estimated fortune of $\$1{,}000{,}000$. This has left two grieving but competing close relatives: Peter, who currently has no wealth, and Paul, who has $\$10{,}000$. With the will missing, they can pursue legal action, but the winner will incur legal costs amounting to 10% of the inheritance. Both Peter and Paul share the same utility function:

$$U(W) = W^{1/2}.$$

Peter and Paul both agree that Peter has a 60% chance of winning the million, while Paul has a 40% chance. The judge cannot issue a split decision; the entire amount must go to one of them.

a. Should Peter and Paul take their dispute to court, or is there a mutually beneficial agreement they could reach instead?

b. Does the conclusion change if the heirs disagree on the probabilities? For instance, what if Peter believes he has an 80% chance of winning, while Paul thinks he has an 80% chance of winning?

Problem 6 (Quadratic Utility and Increasing Risk Aversion)**.** Consider an investor with quadratic utility

$$u(W) = W - \frac{b}{2}W^2, \qquad b > 0, \quad W < \frac{1}{b},$$

where the restriction $W < 1/b$ (the **satiation point**) is needed to keep $u'(W) > 0$.

a. Derive general expressions for *ARA*(W) and *RRA*(W).
b. Take $b = 0.0001$, so satiation occurs at $W = \$10{,}000$. Compute *ARA* and *RRA* at $W = \$1{,}000$ and again at $W = \$4{,}000$.
c. Is quadratic utility DARA or IARA? Contrast your answer with the log-utility investor of Exercise 1, whose willingness to insure the same $\$1{,}000$ gamble *increased* once her wealth fell from $\$5{,}000$ to $\$4{,}000$.

References

Ingersoll, Jonathan E. 1987. *Theory of Financial Decision Making*. Vol. 3. Rowman & Littlefield.