The Capital Asset Pricing Model

Lorenzo Naranjo

August 2026

The Capital Asset Pricing Model

The previous analysis demonstrates that investors with utility functions of the form

$$U = \mu - \frac{1}{2} A \sigma^2$$

will allocate their investments between the tangency portfolio and the risk-free asset.

Since aggregate borrowing equals aggregate lending, all investors collectively hold the tangency portfolio in identical proportions, differing only in the scale of their investments. Consequently, the tangency portfolio represents the **market portfolio**, implying that the market portfolio is an efficient portfolio.

In Figure 1, portfolio A' is efficient, representing a combination of the market portfolio and the risk-free asset. Its return can be expressed as:

$$r_{A'} = (1 - \beta) r_f + \beta r_M.$$

The residual $\varepsilon = r_A - r_{A'}$ is constructed to have a mean of zero, leading to:

$$\mathsf{E}(r_A) = (1 - \beta) r_f + \beta \, \mathsf{E}(r_M).$$

Additionally, the residual is orthogonal to its projection $r_{A'}$, which implies:

$$0 = \mathsf{Cov}(\varepsilon, r_{A'}) = \mathsf{Cov}(r_A - r_{A'}, r_{A'}) = \beta \, \mathsf{Cov}(r_A, r_M) - \beta^2 \, \mathsf{V}(r_M),$$

or equivalently:

$$\beta = \frac{\mathsf{Cov}(r_A, r_M)}{\mathsf{V}(r_M)}.$$

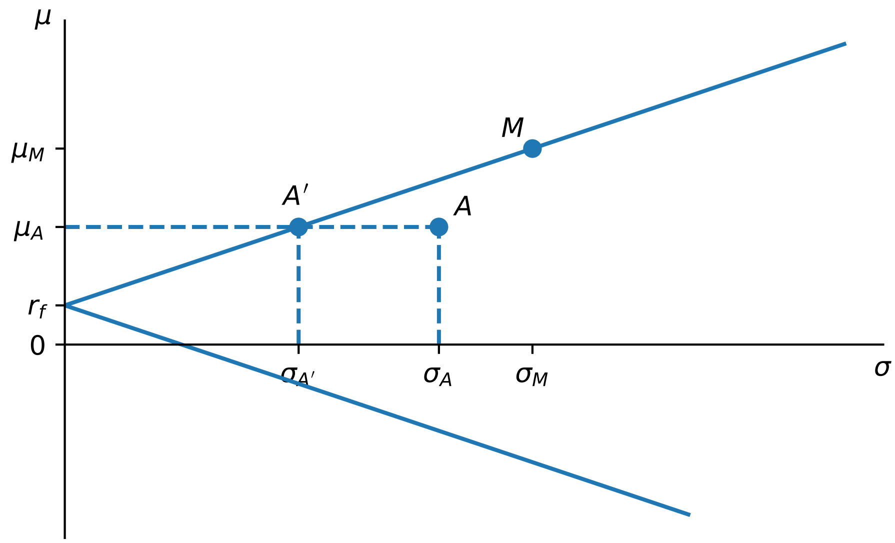

Figure 1: The figure shows the investment opportunity set available to an investor. The upper line determines the efficient frontier whereas the lower line is the set of inefficient portfolios. For a given portfolio A, there is a portfolio A' that has the same expected return as A but the lowest standard deviation. Both A' and M are efficient portfolios.

This formula is commonly used to estimate the beta of an asset. By regressing the returns of the asset r_A on the returns of the market r_M, the slope coefficient obtained corresponds to the ratio of the covariance of r_A with r_M to the variance of r_M.

The CAPM further allows us to decompose returns into systematic and firm-specific components:

$$r_A = (1-\beta)r_f + \beta_A r_M + \varepsilon_A.$$

Here, the residual ε_A represents the idiosyncratic or firm-specific risk, which is diversifiable and orthogonal to market risk. The term $\beta_A r_M$ captures the systematic, non-diversifiable risk of the asset. A higher β indicates greater exposure to market risk.

If the CAPM does not hold, the analysis remains valid by replacing M with the tangency portfolio Q.

Property 1 (The Capital Asset Pricing Model)**.** *The returns of any asset can be decomposed into a systematic component which characterizes the non-diversifiable risk, and a firm-specific* *or idiosyncratic component containing the risk that can be diversified. Thus,*

$$r_A = (1-\beta)r_f + \beta_A r_M + \varepsilon_A, \tag{1}$$

where

$$\beta = \frac{\text{Cov}(r_A, r_M)}{\text{V}(r_M)}.$$

Since the risk in ε_A is not priced, the expected return of the asset depends on how the asset returns covary with the market risk,

$$\text{E}(r_A) = (1-\beta)r_f + \beta\,\text{E}(r_M). \tag{2}$$

Thus, according to the CAPM the only thing that determines the expected return of any risky asset is its covariance with the market portfolio, i.e. its beta.

Example 1 (Computing an Expected Return)**.** If the risk-free rate is 5%, $\beta_{DELL} = 1.3$, and $\text{E}(r_M) = 14\%$, then the CAPM predicts that:

$$\text{E}(r_{DELL}) = 0.05 + 1.3 \times (0.14 - 0.05) = 16.7\%.$$

Dell stock must have an expected annual return of 16.7%.

Note that the CAPM is a *prediction*, and tells us how much the price of the stock should increase on average next year. □

Example 2 (Computing a Stock Beta)**.** Suppose that you know that the correlation between stock A and the market is 0.6. If the standard deviation of A returns is 40% per year, and the standard deviation of the market is 20% per year, the beta of A is

$$\begin{aligned}\beta_A &= \frac{\text{Cov}(r_A, r_M)}{\text{V}(r_M)} = \frac{\sigma_A \sigma_M \rho_{A,M}}{\sigma_M^2} \\ &= \frac{\sigma_A \rho_{A,M}}{\sigma_M} = \frac{0.4 \times 0.6}{0.2} = 1.2.\end{aligned}$$

□

What happens if a security's expected return deviates from the CAPM prediction? One explanation could be market inefficiency, where the stock is mispriced, allowing some investors to exploit this anomaly while the broader market remains indifferent. Alternatively, investors might consider factors beyond mean and variance when selecting portfolios, such as other attributes of returns. In this scenario, the market portfolio would not be efficient, and the CAPM framework would not hold.

Ultimately, distinguishing between these two hypotheses using data alone is impossible. This highlights the dual hypothesis testing problem inherent in finance.

Finally, the return decomposition in equation (1) enables us to break down the variance of any asset into two components:

$$\sigma_A^2 = \beta_A^2 \sigma_M^2 + \sigma^2(\varepsilon_A). \tag{3}$$

Here, σ_A^2 represents the total variance of the asset, which can be divided into systematic variance, $\beta_A^2 \sigma_M^2$, and firm-specific variance, $\sigma^2(\varepsilon_A)$.

Example 3 (Computing the Residual Risk)**.** Suppose that stock A has a standard deviation of returns of 40% per year and a beta of 1.1 with the market. The standard deviation of the market returns is 20%.

This means that the residual variance is

$$\sigma^2(\varepsilon_A) = \sigma_A^2 - \beta_{A,M}^2 \sigma_M^2 = 0.1116.$$

Therefore, the standard deviation of the firm-specific risk is $\sqrt{0.1116} = 33.41\%$. □

References