## **Problem Set 1**

**Instructions**: This problem set is due on Friday 10/31 at 11:59 pm CST and is an individual assignment. All problems must be handwritten. Scan your work and submit a PDF file.

**Problem 1.** You are evaluating the potential acquisition of BayesX Technologies, a company specializing in advanced Bayesian learning algorithms. Analysts at your private equity firm estimate that there is an 80% probability that the firm's value is \$200 million, and a 20% probability that the firm's value is \$50 million.

To refine this analysis, you engage an investment bank to assess the company's value. The bank's valuation process has a 60% probability of correctly identifying the firm's value as high when it is indeed high, and a 40% probability of incorrectly identifying the firm's value as high when it is actually low.

- a. Compute the probability that the company's value is \$200 million given that the bank provides a bullish report.
- b. Calculate the expected value of the company given that the bank provides a bullish report.
- c. Compare the expected value of the company in part b to the unconditional expected value of the company. Explain the difference.
- d. Based on the updated information, determine the maximum amount you are willing to pay for the company.

**Problem 2.** Consider a sequential trade model in which a security has an uncertain value. The value V of the security can either be \$150 with probability 40% or \$250 with probability 60%. The proportion of informed traders is 10%, whereas the proportion of noise traders is 90%. As usual, noise traders buy or sell with equal probability, whereas informed traders only buy when they know the security price is high, and sell when they know the security price is low.

- a. Compute the expected value of the security conditional that the first trade is a sell.
- b. Compute the expected value of the security conditional that the first trade is a buy.
- c. Determine the bid and offer prices of the security that a market maker would post in order to break even on average.
- d. If more traders decide to become informed, should the bid-ask spread increase of decrease? Why?

**Problem 3.** An investor purchased a bond one year ago for \$980. He received \$17 in interest and sold the bond for \$987. What is the holding-period return on his investment?

**Problem 4.** Consider a risky portfolio. The end-of-year cash flow derived from the portfolio will be either \$70,000 or \$200,000 with equal probabilities of 50%. The alternative risk-free investment in T-bills pays 6% per year.

- a. If you require a risk premium of 8% so that the discount rate is 6 + 8 = 14%, how much will you be willing to pay for the portfolio?
- b. Suppose that the portfolio can be purchased for the amount you found in a. What will be the expected rate of return on the portfolio?
- c. Now suppose that you require a risk premium of 12%. What is the price that you will be willing to pay?

**Problem 5.** The stock of company XYZ currently trades at \$100 and just paid a dividend of \$1.8. Suppose that your expectations regarding the stock price and dividends next year are as follows:

State of the Market	Probability	Dividend	Price
Boom	0.3	\$3	\$120
Normal growth	0.5	\$2	\$100
Recession	0.2	\$1	\$80

Compute the mean and standard deviation of the returns for company XYZ.

**Problem 6.** Suppose the economy can only be in one of the following two states: (i) Boom or "good" state and (ii) Recession or "bad" state. Each of the states can occur with an equal probability. At the beginning of a month, you can purchase the following two securities in the market:

- Security 1 is currently trading at \$4. At the end of the month, the security price is expected to increase by \$10 in the good state, and expected to remain unchanged in the bad state.
- Security 2 is currently trading at \$5. At the end of the month, the price of security 2 is expected to remain unchanged in the good state and expected to increase by \$10 in the bad state.
- a. Compute the expected returns of securities 1 and 2.
- b. Compute the standard deviations of returns for securities 1 and 2.
- c. Compute the covariance and the correlation between the returns of two securities.