Derivatives Securities | Prof. Lorenzo Naranjo
FIN 5241 | Fall 2025

Problem Set 2

Instructions: This problem set is due on 12/5 at 11:59 pm CST and is an individual assignment. All problems must be handwritten. Write full sentences to answer questions that require an explanation. Scan your work and submit a PDF file.

Problem 1. Suppose that the FX Trading desk at Goldman Sachs is analyzing a EUR/USD position for a sovereign wealth fund client. The spot price of the Euro (EUR) is USD 1.15 and the EUR/USD exchange rate has a volatility of 4% per annum. The ECB benchmark rate in Europe is 2.25% per year whereas the Fed Funds rate in the United States is 4.50% per year.

a. Calculate the value of a European option to sell EUR 100,000,000 and receive USD 110,000,000 in six months. This represents a notional-weighted strike of 1.10 USD per EUR.
b. Use put-call parity to calculate the price of a European option to buy EUR 100,000,000 for USD 110,000,000 in six months. The client is considering this call as an alternative hedging strategy for their upcoming European acquisition.
c. Explain why the Black-Scholes formula to buy €1 at time T for a predetermined exchange rate K is given by

$$C = Fe^{-rT}\,\Phi(d_1) - Ke^{-rT}\,\Phi(d_2),$$

where $d_1 = \dfrac{\ln(F/K) + \frac{1}{2}\sigma^2 T}{\sigma\sqrt{T}}$, and $d_2 = d_1 - \sigma\sqrt{T}$.

Problem 2. A futures price is currently 70, it's volatility is 30% per year, and the risk-free rate is 5% per year. What is the value of a six-month European call on the futures with a strike price of 72.5?

Problem 3. Calculate the price of a three-month European call option on the spot value of BTC. The three-month futures price is \$86,225, the strike is \$86,000, the risk-free rate is 4.50% and the volatility of the price returns of BTC is 80%.

Problem 4. Briefly answer the following questions.

a. Explain intuitively why the risk-neutral drift of the futures price returns is equal to zero.
b. What does it mean if the theta of an option's position is -0.2 when time is measured in years? If a trader feels that the stock price can change quickly in any direction, what type of option position is appropriate? How the negative theta affects this option position if the trader is wrong?
c. What are the risks for a trader if the gamma of a position is large and positive and the delta is zero?

Problem 5. Consider debit bull and bear spreads with strikes K_1 and $K_2 > K_1$ written on a non-dividend paying asset expiring in a month.

a. In separate diagrams, draw the price, delta and gamma of the bull and the bear spread as a function of the stock price.
b. Determine the sign of the theta if the stock price is equal to K_1 and K_2, respectively.

Problem 6. A financial institution has the following portfolio of over-the-counter options on the British pound.

Type	Position	Delta of option	Gamma of option	Vega of option
Call	-2,000	0.5	0.008	0.40
Call	800	0.8	0.020	0.05
Put	-2,000	-0.4	0.012	0.12
Put	1,500	-0.6	0.007	0.42

A traded option is available with a delta of -0.4, a gamma of 0.011, and a vega of 0.08. Please answer the following:

a. What position in the traded option and the euro, would make the portfolio both gamma neutral and delta neutral?
b. What position in the traded option and the euro, would make the portfolio both vega neutral and delta neutral?

Problem 7. Which of the following options strategies have **negative theta**? Assume that all options are European and written on an asset that does not pay dividends and that the risk-free rate is positive. Briefly explain why.

a. A short straddle with a strike price equal to the current stock price.
b. A long strangle with strike prices $K_1 < K_2$ in which the current stock price is between K_1 and K_2.
c. A bull spread with strike prices $K_1 < K_2$ in which the current stock price is more than K_2.
d. A bear spread with strike prices $K_1 < K_2$ in which the current stock price is more than K_2.

Problem 8. Suppose that the derivatives desk at Morgan Stanley has just sold 10,000 European puts to BlackRock. Each put is written on the MS-30 Tech Index, which tracks 30 high-growth technology companies. The index is currently at 4,500 points, and pays a dividend yield of 2% per year. The puts expire in one year, have a strike price of 4,100 and are cash settled at expiration. The risk-free rate is 4.5% per year with continuous compounding. The volatility desk estimates that the volatility of the index returns is 45% and expected to remain constant for the next year.

a. There's an ETF (ticker: MSTX) that tracks the index perfectly and currently trades for $130. How many shares of the ETF does the trader need to buy/sell initially in order to hedge the exposure created by the sale of the puts?
b. How much money does the trader need to borrow/lend today in order to make sure that the strategy is self-financing?
c. When hedging the puts, should the trader be more worried about gamma or vega risk?

Problem 9. Consider a blue-chip tech stock in JP Morgan's equity derivatives portfolio that pays a dividend yield of 2% and has a volatility of returns of 45%. The stock price is \$95 and the risk-free rate is 4.5%.

a. Compute the price of an asset-or-nothing put that pays 1 share of the stock if the stock price in one month is below \$90. This exotic option was requested by a hedge fund client looking to implement a sophisticated collar strategy.
b. Compute the price of a cash-or-nothing put that pays \$100 if the stock price in one month is below \$90. The trading desk is considering offering this binary option to complement the client's existing positions.

Problem 10. Consider a non-dividend paying stock that trades for \$50. Every 3-months, the stock price can increase or decrease by 10%. The risk-free rate is 5% per year with continuous compounding. Compute the price of the following path-dependent options expiring in 6 months.

a. A floating lookback call that pays $S_T - S_{min}$ at maturity.
b. A floating lookback put that pays $S_{max} - S_T$ at maturity.
c. An average price Asian put option that pays $\max(50 - \bar{S}, 0)$ at maturity.
d. An average strike Asian call option that pays $\max(S_T - \bar{S}, 0)$ at maturity.

Normal Distribution Table

The table below computes $\Phi(z) = \mathrm{P}(Z \leq z)$ if $Z \sim \mathcal{N}(0,1)$ and $z \geq 0$. The rows denote the first decimal whereas the columns denote the second decimal. If you need $\Phi(-z)$, you can always use $\Phi(-z) = 1 - \Phi(z)$.

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990