Problem Set 1

Instructions: This problem set is due on 9/5 at 11:59 pm CST and is an individual assignment. All problems must be handwritten. Scan your work and submit a PDF file.

Problem 1. Suppose your production opportunity set in a world with perfect certainty consists of the following investment projects:

Project	Maximum Investment	IRR
A	$1,000,000	8%
B	$1,000,000	25%
C	$2,000,000	4%
D	$3,000,000	40%

Consider an investor with utility function $U(C_0, C_1) = C_0 + 0.9C_1$. This investor has $7,000,000 in wealth

a. If the investor has no access to capital markets.
   i. What is the value of her portfolio of projects?
   ii. How much should she consume today and how much next period?

b. Suppose now that a new bank comes to town, that allows the investor to borrow or lend at the interest rate of 10% per period.
   i. What is the value of her portfolio of projects?
   ii. How much should she consume today and how much next period?

c. If the bank was forced to close, how much the government would have to pay the investor to leave her indifferent?

Problem 2. Consider an investor with utility $U(C_0, C_1) = \ln(C_0) + 2\ln(C_1)$ for consumption today and next period. The investor has initial wealth $W = 1$ and can invest $K = 1 - C_0$ in a technology that produces $f(K) = \sqrt{K}$ next period.

a. Compute the optimal consumption at dates 0 and 1.

Assume now that the investor has access to capital markets and can borrow or lend at r.

b. Compute the present value and the net present value of the technology.
c. If the investor sells the company at the value computed in b., what is her optimal consumption now?

Problem 3. You are offered the possibility to participate at the following lottery:

Gain	Probability
2	0.5
0	0.5

The cost at participating at the lottery is 1 unit of consumption. If you chose not to participate, you keep your unit of consumption.

a. Is this a fair gamble?
b. Show that the decision not to participate at the gamble implies that your utility function is concave.

Problem 4. Consider gamble A:

Gain	Probability
-2	0.09
4	0.30
10	0.40
16	0.21

From A, we can construct another gamble B by adding white noise to a number of outcomes. Indeed, we can replace the outcome 4 by the gamble A':

- 3 with probability 1/2
- 5 with probability 1/2

with $\mathrm{E}(A') = 4$. In the same manner, we can replace outcome 16 with gamble A'':

- 12 with probability 1/3
- 18 with probability 2/3

with $\mathrm{E}(A'') = 16$.

Show formally why any risk averse individual prefers gamble A to gamble B.

Problem 5. A distant relative in Europe has recently passed away, leaving behind an estimated fortune of $1 million. This has left two grieving but competing close relatives: Peter, who currently has no wealth, and Paul, who has $10,000. With the will missing, they can pursue legal action, but the winner will incur legal costs amounting to 10% of the inheritance. Both Peter and Paul share the same utility function:

$$U(W) = W^{1/4}$$

Peter and Paul both agree that Peter has a 60% chance of winning the million, while Paul has a 40% chance. The judge cannot issue a split decision; the entire amount must go to one of them.

a. Should Peter and Paul take their dispute to court, or is there a mutually beneficial agreement they could reach instead?
b. Does the conclusion change if the heirs disagree on the probabilities? For instance, what if Peter believes he has an 80% chance of winning, while Paul thinks he has an 80% chance of winning?