Problem Set 6

Instructions: This problem set is due on 4/14 at 11:59 pm CST and is an individual assignment. All problems must be handwritten. Scan your work and submit a PDF file. For all probability computations, use the table at the end of this assignment.

Note: All interest rates and dividend yields are expressed per year with continuous compounding.

Problems

Problem 1. What is the price of a European put option on a non-dividend-paying stock when the stock price is $88, the strike price is $90, the risk-free interest rate is 5% per annum, the volatility is 45% per annum, and the time to maturity is eight months?

Problem 2. Consider an option on a non-dividend-paying stock when the stock price is $50, the exercise price is $49, the risk-free interest rate is 4.5%, the volatility is 38% per annum, and the time to maturity is three months.

a. What is the price of the option if it is a European call?
b. What is the price of the option if it is an American call?
c. What is the price of the option if it is a European put?
d. Verify that put-call parity holds.

Problem 3. Consider a European call option expiring in 6 months and with strike price equal to $48 on a non-dividend paying stock that currently trades for $50. Interestingly, the volatility of the stock is zero. If the risk-free rate is 8% per year with continuous compounding, what is the price of the option?

Problem 4. Consider a European call option expiring in 6 months and with strike price equal to $52 on a non-dividend paying stock that currently trades for $50. Interestingly, the volatility of the stock is zero. If the risk-free rate is 5% per year with continuous compounding, what is the price of the option?

Problem 5. Consider a European put option expiring in 9 months and strike price $152 written on a non-dividend paying stock. The risk-free rate is 6% per year with continuous compounding and the stock price is $150. What is the minimum price for the put that would allow you to compute its implied volatility?

Problem 6. Suppose that the sales team of a trading desk just sold a European call option contract, that is 100 European call options, to an important client. The contract is written on a non-dividend paying stock that trades for $150, expires in two years and has a strike price of $155. The risk-free rate is 5% per year with continuous compounding. A trader of the desk estimate that the volatility of the stock returns is 55% and expected to remain constant for the life of the contract.

a. How many shares of the stock does the trader need to buy/sell initially in order to hedge the exposure created by the sale of the contract?
b. How many risk-free bonds with face value $155 and expiring in two years does the trader need to buy/sell in order to make sure that the strategy is self-financing?
c. How many risk-free bonds with face value $100 and expiring in two years does the trader need to buy/sell in order to make sure that the strategy is self-financing?
d. Why choosing a different face value for the bonds does not change the price of the call option.

Problem 7. A call option on a non-dividend-paying stock has a market price of $12.39. The stock price is $100, the exercise price is $100, the time to maturity is six months, and the risk-free interest rate is 5% per annum. What is the implied volatility of the call? Explain the method you used to find the implied volatility.

Problem 8. Calculate the value of a three-month at-the-money European call option on a stock index when the index is at 5,000, the risk-free interest rate is 5% per annum, the volatility of the index is 30% per annum, and the dividend yield on the index is 2% per annum.

Problem 9. The S&P 100 index currently stands at 2,392 and has a volatility of 50% per annum. The risk-free rate of interest is 4% per annum and the index provides a dividend yield of 1% per annum. Calculate the value of a three-month European put with strike price 2,300.

The Standard Normal Distribution

The following table reports values for $\phi(z) = P(Z \leq z)$, where $Z \sim \mathcal{N}(0,1)$.

z	P(Z ≤ z)	z	P(Z ≤ z)	z	P(Z ≤ z)	z	P(Z ≤ z)
-2.37	0.0089	-1.17	0.1210	0.03	0.5120	1.23	0.8907
-2.32	0.0102	-1.12	0.1314	0.08	0.5319	1.28	0.8997
-2.27	0.0116	-1.07	0.1423	0.13	0.5517	1.33	0.9082
-2.22	0.0132	-1.02	0.1539	0.18	0.5714	1.38	0.9162
-2.17	0.0150	-0.97	0.1660	0.23	0.5910	1.43	0.9236
-2.12	0.0170	-0.92	0.1788	0.28	0.6103	1.48	0.9306
-2.07	0.0192	-0.87	0.1922	0.33	0.6293	1.53	0.9370
-2.02	0.0217	-0.82	0.2061	0.38	0.6480	1.58	0.9429
-1.97	0.0244	-0.77	0.2206	0.43	0.6664	1.63	0.9484
-1.92	0.0274	-0.72	0.2358	0.48	0.6844	1.68	0.9535
-1.87	0.0307	-0.67	0.2514	0.53	0.7019	1.73	0.9582
-1.82	0.0344	-0.62	0.2676	0.58	0.7190	1.78	0.9625
-1.77	0.0384	-0.57	0.2843	0.63	0.7357	1.83	0.9664
-1.72	0.0427	-0.52	0.3015	0.68	0.7517	1.88	0.9699
-1.67	0.0475	-0.47	0.3192	0.73	0.7673	1.93	0.9732
-1.62	0.0526	-0.42	0.3372	0.78	0.7823	1.98	0.9761
-1.57	0.0582	-0.37	0.3557	0.83	0.7967	2.03	0.9788
-1.52	0.0643	-0.32	0.3745	0.88	0.8106	2.08	0.9812
-1.47	0.0708	-0.27	0.3936	0.93	0.8238	2.13	0.9834
-1.42	0.0778	-0.22	0.4129	0.98	0.8365	2.18	0.9854
-1.37	0.0853	-0.17	0.4325	1.03	0.8485	2.23	0.9871
-1.32	0.0934	-0.12	0.4522	1.08	0.8599	2.28	0.9887
-1.27	0.1020	-0.07	0.4721	1.13	0.8708	2.33	0.9901
-1.22	0.1112	-0.02	0.4920	1.18	0.8810	2.37	0.9911