Options, Futures and Derivative Securities FIN 451
Time: 80 minutes 20 points

Mock Midterm 3

Questions

Problem 1 (3 pts). Suppose that the derivatives desk at Morgan Stanley has just sold 10,000 European puts to BlackRock. Each put is written on the MS-30 Tech Index, which tracks 30 high-growth technology companies. The index is currently at 4,500 points, and pays a dividend yield of 2% per year. The puts expire in one year, have a strike price of 4,100 and are cash settled at expiration. The risk-free rate is 4.5% per year with continuous compounding. The volatility desk estimates that the volatility of the index returns is 45% and expected to remain constant for the next year.

a. There's an ETF (ticker: MSTX) that tracks the index perfectly and currently trades for $130. How many shares of the ETF does the trader need to buy/sell initially in order to hedge the exposure created by the sale of the puts?

b. How much money does the trader need to borrow/lend today in order to make sure that the strategy is self-financing?

c. When hedging the puts, should the trader be more worried about gamma or vega risk?

a. $$d_1 = \frac{\ln(4500/4100) + (0.045 - 0.02 + \frac{1}{2}0.45^2)\times 1}{0.45\sqrt{1}}$$

$$= 0.487$$

$$d_2 = 0.487 - 0.45\sqrt{1} = 0.037$$

$$\Phi(-d_1) \simeq 0.3192 \qquad \Phi(-d_2) = 0.4920$$

$$N_S = -10{,}000 \times \frac{4{,}500}{130} \times 0.3192\, e^{-0.02\times 1} = -108{,}304.41$$

The trader needs to sell 108,304 shares of the ETF.

b. The trader needs to lend

$$10{,}000 \times 4{,}100 \times e^{-0.045\times 1} \times 0.4920$$
$$= \$19{,}284{,}381$$

to make the strategy self-financing.

c. Since the option has one year to maturity, the trader should be more worried about vega risk. Gamma risk is more important for very short maturity options.

Problem 2 (3 pts). Suppose that the FX Trading desk at Goldman Sachs is analyzing a EUR/USD position for a sovereign wealth fund client. The spot price of the Euro (EUR) is USD 1.15 and the EUR/USD exchange rate has a volatility of 4% per annum. The ECB benchmark rate in Europe is 2.25% per year whereas the Fed Funds rate in the United States is 4.50% per year.

a. Calculate the value of a European option to sell EUR 100,000,000 and receive USD 110,000,000 in six months. This represents a notional-weighted strike of 1.10 USD per EUR.
b. Use put-call parity to calculate the price of a European option to buy EUR 100,000,000 for USD 110,000,000 in six months. The client is considering this call as an alternative hedging strategy for their upcoming European acquisition.
c. Explain why the Black-Scholes formula to buy €1 at time T for a predetermined exchange rate K is given by

$$C = Fe^{-rT}\,\Phi(d_1) - Ke^{-rT}\,\Phi(d_2),$$

where $d_1 = \dfrac{\ln(F/K) + \frac{1}{2}\sigma^2 T}{\sigma\sqrt{T}}$, and $d_2 = d_1 - \sigma\sqrt{T}$.

Note: The table below might come handy to compute $\Phi(-d_1)$ and $\Phi(-d_2)$.

z	P(Z ≤ z)	z	P(Z ≤ z)	z	P(Z ≤ z)	z	P(Z ≤ z)
-2.00	0.0228	-1.97	0.0244	-1.94	0.0262	-1.91	0.0281
-1.99	0.0233	-1.96	0.0250	-1.93	0.0268	-1.90	0.0287
-1.98	0.0239	-1.95	0.0256	-1.92	0.0274	-1.89	0.0294

a. The implicit strike price of the currency option is \$1.10 per €.

$$d_1 = \frac{\ln(1.15/1.10) + (0.0450 - 0.0225 + \tfrac{1}{2}\,0.04^2) \times 6/12}{0.04\sqrt{6/12}}$$

$$= 1.983$$

$$d_2 = 1.983 - 0.04\sqrt{6/12} = 1.954$$

$\phi(-d_1) = 0.0239 \qquad \phi(-d_2) = 0.0256$

$$\text{Put} = 100{,}000{,}000 \times \left(1.10\, e^{-0.045 \times 6/12} \times 0.0256 - 1.15\, e^{-0.0225 \times 6/12} \times 0.0239\right)$$

$$= \$38{,}989.90$$

b. $$\text{Call} = 38{,}989.90 + 100{,}000{,}000 \times \left(1.15\, e^{-0.0225 \times 6/12} - 1.10\, e^{-0.045 \times 6/12}\right)$$

$$= \$6{,}199{,}853.94.$$

c. We can always write

$$d_1 = \frac{\ln(S/K) + (r - r^* + \tfrac{1}{2}\sigma^2)T}{\sigma\sqrt{T}}$$

$$= \frac{\ln(S e^{(r-r^*)T}/K) + \tfrac{1}{2}\sigma^2 T}{\sigma\sqrt{T}}$$

$$= \frac{\ln(F/K) + \tfrac{1}{2}\sigma^2 T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

$$c = S e^{-r^* T}\phi(d_1) - K e^{-rT}\phi(d_2)$$

$$= S e^{(r-r^*)T}\phi(d_1) e^{-rT} - K e^{-rT}\phi(d_2)$$

$$= F e^{-rT}\phi(d_1) - K e^{-rT}\phi(d_2)$$

where $F = S e^{(r-r^*)T}$ is the forward price.

Problem 3 (2 pts). Consider a European put option expiring in 6 months and with strike price equal to $103, written on a stock that currently trades for $100. Interestingly, the volatility of the stock is zero. The risk-free rate is 5% per year with continuous compounding and the stock pays a dividend yield of 2% per year.

a. Compute the price of the put option.
b. Does put-call parity hold if the volatility of the asset returns is equal to zero?

a. $\text{Put} = \max(103e^{-0.05 \times 6/12} - 100e^{-0.02 \times 6/12}, 0)$

$= \$1.45$

b. Put-call parity holds regardless of the volatility of the stock. Put-call parity only depends on the absence of arbitrage opportunities.

Thus,

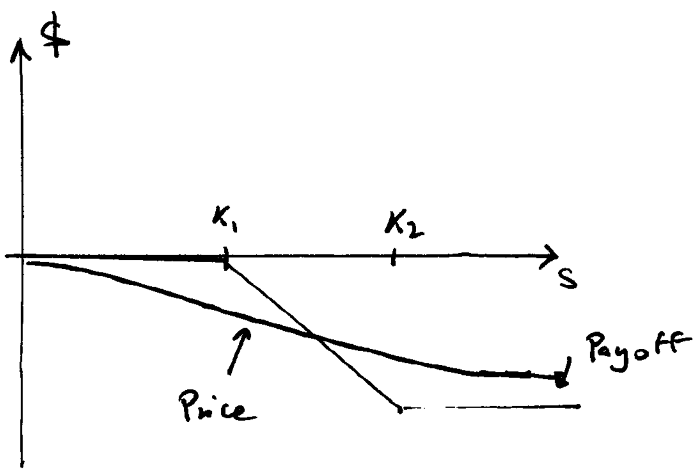
$\text{Call} = 1.45 - 103e^{-0.05 \times 6/12} + 100e^{-0.02 \times 6/12} = \$0.$

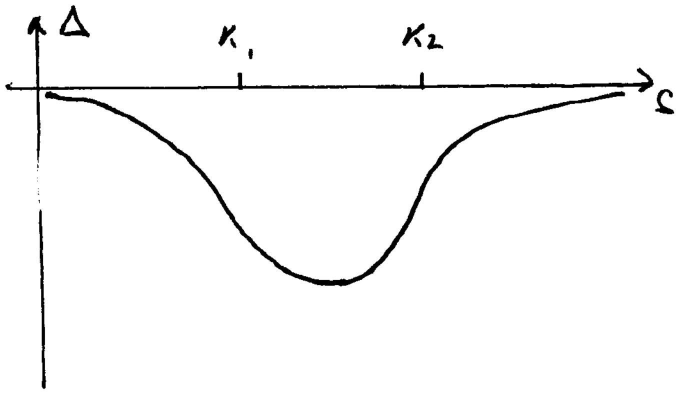
Problem 4 (2 pts). Consider a credit bear spread with strikes K_1 and $K_2 > K_1$ made using European call options written on a non-dividend paying asset and expiring in two months.

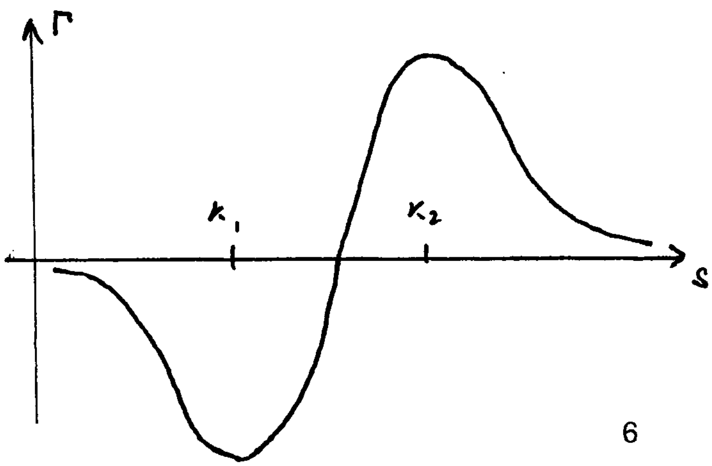
a. In separate diagrams, draw the price, delta and gamma of the bear spread as a function of the stock price.

b. Determine the sign of the theta if the stock price is equal to K_1 and K_2, respectively.

a.







b. At K_1 the payoff is concave, so the $\Gamma < 0$ and $\Theta > 0$

At K_2 the payoff is convex, so $\Gamma > 0$ and $\Theta < 0$

Problem 5 (2 pts). Consider a blue-chip tech stock in JP Morgan's equity derivatives portfolio that pays a dividend yield of 2% and has a volatility of returns of 45%. The stock price is \$95 and the risk-free rate is 4.5%.

a. Compute the price of an asset-or-nothing put that pays 1 share of the stock if the stock price in one month is below \$90. This exotic option was requested by a hedge fund client looking to implement a sophisticated collar strategy.
b. Compute the price of a cash-or-nothing put that pays \$100 if the stock price in one month is below \$90. The trading desk is considering offering this binary option to complement the client's existing positions.

a. $$d_1 = \frac{\ln(95/90) + (0.045 - 0.02 + \frac{1}{2}\, 0.45^2) \times \frac{1}{12}}{0.45\sqrt{1/12}}$$

$$= 0.497 \quad \Rightarrow \quad \phi(-d_1) = 0.3015$$

Asset or nothing put $= 95\, e^{-0.02 \times 1/12} \times 0.3015$

$= \$23.59$

b. $d_2 = d_1 - \sigma\sqrt{T} = 0.497 - 0.45\sqrt{1/12} = 0.367$

$\phi(-d_2) = 0.3557$

Cash or nothing put $= 100\, e^{-0.045 \times 1/12} \times 0.3557$

$= \$35.44$.

Problem 6 (2 pts). Calculate the price of a three-month European put option on Bitcoin futures expiring in three months. The three-month futures price is \$89,215, the strike is \$89,000, the risk-free rate is 4.50% and the volatility of the price returns of BTC is 85%.

$$d_1 = \frac{\ln(89215/89000) + \frac{1}{2}\, 0.85^2 \times 3/12}{0.85\sqrt{3/12}}$$

$$= 0.218$$

$$\phi(-d_1) = 0.4129$$

$$d_2 = 0.218 - 0.85\sqrt{3/12} = -0.207$$

$$\phi(-d_2) = 0.5910$$

$$\text{Put} = 89000\, e^{-0.045 \times 3/12} \times 0.5910$$

$$- 89215\, e^{-0.045 \times 3/12} \times 0.4129$$

$$= \$15{,}585.80$$

Problem 7 (2 pts). Determine whether the following statements are true or false and briefly explain why.

a. A chooser option is very similar to a straddle since at the moment in which you can choose whether you want a call or a put you get pretty much what a straddle pays off.
b. In order to be able to price a forward-start option in closed-form it is crucial that the option starts at-the-money.

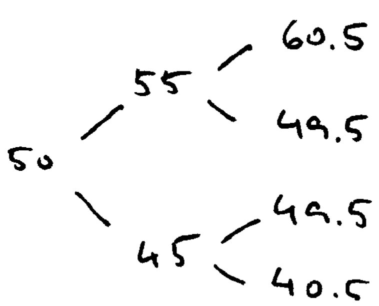
a. TRUE : When buying both a call and a put, the straddle is giving the option to choose whether you will exercise the call or the put, so both products are very similar.

b. TRUE : To price the forward starting option we need to compute the expected price of the option when it is starting. By making the option ATM, the value of d_1 becomes deterministic and therefore goes out of the expectation.

Problem 8 (4 pts). Consider a non-dividend paying stock that trades for \$50. Every 3-months, the stock price can increase or decrease by 10%. The risk-free rate is 5% per year with continuous compounding. Compute the price of the following path-dependent options expiring in 6 months.

a. A floating lookback call that pays $S_T - S_{min}$ at maturity.
b. A floating lookback put that pays $S_{max} - S_T$ at maturity.
c. An average price Asian put option that pays $\max(50 - \bar{S}, 0)$ at maturity.
d. An average strike Asian call option that pays $\max(S_T - \bar{S}, 0)$ at maturity.

$$q = \frac{e^{0.05 \times 3/12} - 0.9}{1.1 - 0.9} = 0.563 \qquad 1 - q = 0.437$$



S_{max}	S_{min}	$\bar{S}$
60.5	50	55.16
55	49.5	51.50
50	45	48.16
50	40.5	45.16

$S_T - S_{min}$	$S_{max} - S_T$	$\max(50 - \bar{S}, 0)$	$\max(S_T - \bar{S}, 0)$
10.5	0	0	5.34
0	5.5	0	0
4.5	0.5	1.84	1.34
0	9.5	4.84	0

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a. $\text{Price} = \left(10.5\, q^2 + 4.5\, q(1-q)\right) e^{-0.05 \times 6/12} = \$4.33$

b. $\text{Price} = \left((5.5 + 0.5)\, q\,(1-q) + 9.5(1-q)^2\right) e^{-0.05 \times 6/12}$

$= \$3.21$

c. $\text{Price} = \left(1.84\, q\,(1-q) + 4.84\,(1-q)^2\right) e^{-0.05 \times 6/12}$

$= \$1.34$

d. $\text{Price} = \left(5.34\, q^2 + 1.34\, q(1-q)\right) e^{-0.05 \times 6/12}$

$= \$1.97$