

Mock Midterm 2

Questions

Problem 1 (3 pts). A non-dividend paying stock price is currently \$120. It is known that at the end of 8 months it will be either \$150 or \$90. The risk-free interest rate is 10% per annum with continuous compounding. What is the value of an 8-month European call option with a strike price of \$125? Use the replicating portfolio argument and indicate the number of shares required to hedge the position. Use a risk-free bond with face value \$100 in your computations.

Problem 2 (3 pts). A stock price is currently \$50. It is known that at the end of 4 months it will be either \$60 or \$40. Analysts at a prestigious bank have determined that there is a 70% probability that the stock will be \$60 in four months. The risk-free interest rate is 8% per annum with continuous compounding. The stock pays a dividend yield of 4% per year.

- a. What is the value of a 4-month European put option with a strike price of \$52? Use the risk-neutral probability method to compute the value of the option.
- b. Using put-call parity, determine the price of a European call with the same characteristics.

Problem 3 (2 pts). The stock of Boulder Corp. trades at \$130 per share and pays a dividend yield of 10% per year with continuous compounding. The stock could trade at \$160 or \$100 in six months, depending on how the economy evolves. The risk-free rate is currently 5% per year. A banker just sold to a client an instrument that pays in six months \$5,000 if the stock price goes up and \$1,000 otherwise. How many shares of the stock (per instrument sold) does she need to buy or sell to hedge her exposure?

Problem 4 (2 pts). A stock that pays a dividend yield of 4% per year costs \$200. Financial analysts at all major investment banks agree there is a 60% probability that the stock will trade for \$250 next year if the company succeeds in developing a novel AI algorithm. Otherwise, the stock price could fall to \$100. The risk-free rate is 4% per year and is expected to remain constant. What should be the price of a security that pays \$0 next year if the stock goes up and \$1,000 otherwise?

Problem 5 (4 pts). The current price of a non-dividend-paying stock is \$250. Every three months, it is expected to increase or decrease by 20%. The risk-free rate is 10% per year. Compute the price of an American put option with a strike price of \$255 and a maturity of six months written on the stock. In each node, explain why you would exercise the put or not.

Problem 6 (2 pts). *A friend of yours, who is attending a prestigious university, says: “I learned in class that you should never exercise early an American call option written on a non-dividend paying asset. On the other hand, an American put option with similar characteristics should be exercised early if the stock price is low enough”* Determine whether the statement is true or false.

Problem 7 (4 pts). Let S be the price of GOOG stock that follows a geometric Brownian motion such that $dS = \mu S dt + \sigma S dW$. Your sales team would like to launch a new product called PWR that tracks the price of GOOG to the power 1.5. Traders at your bank are excited but would like to know the dynamics of $Y = S^{1.5}$, since they would also like to write options on PWR.

- a. Write down the dynamics of Y , and determine its drift and volatility.
- b. If GOOG stock is currently 160, compute the probability that PWR is higher than 2,220 in six months. In your computations, use $\mu = 0.15$ and $\sigma = 0.50$.

Formula Sheet

In the following, S denotes the stock or spot price of an asset, r is the continuously-compounded risk-free rate expressed per year, δ denotes the dividend yield, T denotes the time-to-maturity of a forward, futures or an option, and K denotes the strike price of an option or the delivery price of a forward contract.

Binomial Pricing

At any node of a binomial tree in which the stock price can move up to $S_u = u \times S$ or down to $S_d = d \times S$, the risk-neutral probability of an up-move is given by

$$q = \frac{Se^{(r-q)\Delta t} - S_d}{S_u - S_d} = \frac{e^{(r-q)\Delta t} - d}{u - d},$$

where Δt denotes the length of each period. To make the tree consistent with the observed volatility of stock returns, we typically choose $u = e^{\sigma\sqrt{\Delta t}}$ and $d = 1/u$.

Impact of Dividends

For European call and put options with strike price K and time-to-expiration T written on a non-dividend paying asset, we have that

$$C - P = Se^{-\delta T} - Ke^{-rT},$$

where C and P denote the call and put prices. Put-call parity implies the following bounds for European call and put options:

$$\begin{aligned} \max(Se^{-\delta T} - Ke^{-rT}, 0) &\leq C \leq S, \\ \max(Ke^{-rT} - Se^{-\delta T}, 0) &\leq P \leq Ke^{-rT}. \end{aligned}$$

Modeling Stock Prices

Stock prices follow a geometric Brownian motion such that

$$\frac{dS}{S} = \mu dt + \sigma dW, \quad (1)$$

where μ denotes the drift, σ the instantaneous volatility of stock returns and W is a standard Brownian motion. If we take $Y = f(S)$ where f is a twice-differentiable function, then Ito's lemma implies

$$dY = Y_S dS + \frac{1}{2} Y_{SS} (dS)^2,$$

where $Y_S = f'(S)$ and $Y_{SS} = f''(S)$. We showed in class that $(dS)^2 = \sigma^2 S^2 dt$.

Equation (1) implies that $\ln(S_T)$ is normally distributed with mean $\ln(S_0) + \left(\mu - \frac{1}{2}\sigma^2\right)T$ and standard deviation $\sigma\sqrt{T}$.

The Standard Normal Distribution

The following table reports values for $\phi(z) = P(Z \leq z)$, where $Z \sim \mathcal{N}(0, 1)$.

z	$P(Z \leq z)$	z	$P(Z \leq z)$	z	$P(Z \leq z)$	z	$P(Z \leq z)$
-2.37	0.0089	-1.17	0.1210	0.03	0.5120	1.23	0.8907
-2.32	0.0102	-1.12	0.1314	0.08	0.5319	1.28	0.8997
-2.27	0.0116	-1.07	0.1423	0.13	0.5517	1.33	0.9082
-2.22	0.0132	-1.02	0.1539	0.18	0.5714	1.38	0.9162
-2.17	0.0150	-0.97	0.1660	0.23	0.5910	1.43	0.9236
-2.12	0.0170	-0.92	0.1788	0.28	0.6103	1.48	0.9306
-2.07	0.0192	-0.87	0.1922	0.33	0.6293	1.53	0.9370
-2.02	0.0217	-0.82	0.2061	0.38	0.6480	1.58	0.9429
-1.97	0.0244	-0.77	0.2206	0.43	0.6664	1.63	0.9484
-1.92	0.0274	-0.72	0.2358	0.48	0.6844	1.68	0.9535
-1.87	0.0307	-0.67	0.2514	0.53	0.7019	1.73	0.9582
-1.82	0.0344	-0.62	0.2676	0.58	0.7190	1.78	0.9625
-1.77	0.0384	-0.57	0.2843	0.63	0.7357	1.83	0.9664
-1.72	0.0427	-0.52	0.3015	0.68	0.7517	1.88	0.9699
-1.67	0.0475	-0.47	0.3192	0.73	0.7673	1.93	0.9732
-1.62	0.0526	-0.42	0.3372	0.78	0.7823	1.98	0.9761
-1.57	0.0582	-0.37	0.3557	0.83	0.7967	2.03	0.9788
-1.52	0.0643	-0.32	0.3745	0.88	0.8106	2.08	0.9812
-1.47	0.0708	-0.27	0.3936	0.93	0.8238	2.13	0.9834
-1.42	0.0778	-0.22	0.4129	0.98	0.8365	2.18	0.9854
-1.37	0.0853	-0.17	0.4325	1.03	0.8485	2.23	0.9871
-1.32	0.0934	-0.12	0.4522	1.08	0.8599	2.28	0.9887
-1.27	0.1020	-0.07	0.4721	1.13	0.8708	2.33	0.9901
-1.22	0.1112	-0.02	0.4920	1.18	0.8810	2.37	0.9911