Prof. Lorenzo Naranjo Spring 2025

Options on Currencies

Exchange Rates

The (nominal) exchange rate between two currencies is the number of domestic currency units per unit of foreign currency. We need to be careful, though, since the street market convention for the EUR/USD exchange rate implies:

- The *quote* currency is the US dollar (USD)
- The *base* currency is the Euro (EUR)

For example, the direct quotation of the EUR/USD could be $1.4380/€, and represents the price in USD of 1 EUR. Note that you could always define it the other way around (indirect-quotes), and this is done for many currency pairs as well.

The market convention of calling this exchange rate EUR/USD might be misleading. It is written EUR/USD, EUR-USD or EURUSD but it really represents the number of USD per EUR, i.e. $\$1.4380 \Leftrightarrow €1$. Be careful, though, as in some textbooks you might find it the other way around.

Example 1. If the EUR/USD exchange rate is $1.47/€, for a US investor, 1 Euro is worth $1.47, but in Europe, how many Euros is worth $1?

$$\$1 = \frac{1}{1.47} = €0.68/\$.$$

The exchange rate is a *relative price*.

Some currency pairs such as EUR/USD or GBP/USD use the USD as the quote currency. However, most currency pairs are expressed using the dollar as the base currency, i.e., USD/JPY, USD/CNY, USD/CLP, etc.

The Risk-Neutral Process for a Currency

Let's denote by r the domestic risk-free rate and by r^* the foreign currency risk-free rate. The risk-neutral process for an exchange-rate S expressed with direct-quotes is then:

$$dS = (r - r^*)Sdt + \sigma S dW^*$$

Using the previous notation, the forward price with maturity T for the currency is:

$$F = Se^{(r-r^*)T}$$

Example 2. The EUR/USD currently trades at \$1.18663. The continuously compounded 9-month risk-free rates in USD and EUR are 1.5% and 0.5% per year, respectively. The 9-month EUR/USD forward rate is then:

$$F = 1.18663e^{(0.015-0.005)(9/12)} = \$1.19556$$

or +89.3 forward-points.

Options on Currencies

Options on currencies reveal an interesting relationship between the underlying asset and the numeraire used to express the price of the asset. Consider an American investor analyzing a *call* option on the EUR/USD with maturity 1-year, strike price \$1.25 over a notional of €1 million. From the point of the view of a European investor, that option is really a *put* on the USD/EUR with same maturity, strike price €0.80 over a notional of \$1.25 million. Hence, it is convenient to be explicit about the currency being bought and the one being sold when specifying the contract, i.e., we will talk about a **EUR call / USD put** when describing the previous contract.

Black-Scholes Model for Currencies

It is common to express the Black-Scholes formulas for options on currencies as a function of the corresponding forward price:

$$C = Fe^{-rT}\,\Phi(d_1) - Ke^{-rT}\,\Phi(d_2)$$
$$P = Ke^{-rT}\,\Phi(-d_2) - Fe^{-rT}\,\Phi(-d_1)$$

where

$$F = Se^{(r-r^*)T},\ d_1 = \frac{\ln(F/K) + \frac{1}{2}\sigma^2 T}{\sigma\sqrt{T}} \text{ and } d_2 = d_1 - \sigma\sqrt{T}.$$

An option with a strike price equal to its corresponding forward price is called at-the-money-forward (ATMF). Remember put-call parity for currencies:

$$C - P = Se^{-r^*T} - Ke^{-rT}$$

When $K = F = Se^{(r-r^*)T}$ we have that $C - P = 0$, i.e., when the strike price is equal to the forward price a call and a put with the same maturity are worth the same.

Practice Problems

Solutions to all problems can be found at lorenzonaranjo.com/fin451.

Problem 1. Calculate the value of an eight-month European put option on a currency with a strike price of 0.50. The current exchange rate is 0.52, the volatility of the exchange rate is 12%, the domestic risk-free interest rate is 4% per annum, and the foreign risk-free interest rate is 8% per annum.

Problem 2. Suppose that the spot price of the Canadian dollar (CAD) is USD $0.75 and that the CAD/USD exchange rate has a volatility of 4% per annum. The risk-free rates of interest in Canada and the United States are 9% and 7% per annum, respectively. Calculate the value of a European call option to buy one Canadian dollar for USD $0.75 in nine months. Use put-call parity to calculate the price of a European put option to sell one Canadian dollar for U.S. $0.75 in nine months.