

Mock Midterm 3

Questions

Problem 1 (3 pts). Suppose that the derivatives desk at Morgan Stanley has just sold 10,000 European puts to BlackRock. Each put is written on the MS-30 Tech Index, which tracks 30 high-growth technology companies. The index is currently at 4,500 points, and pays a dividend yield of 2% per year. The puts expire in one year, have a strike price of 4,100 and are cash settled at expiration. The risk-free rate is 4.5% per year with continuous compounding. The volatility desk estimates that the volatility of the index returns is 45% and expected to remain constant for the next year.

- There's an ETF (ticker: MSTX) that tracks the index perfectly and currently trades for \$130. How many shares of the ETF does the trader need to buy/sell initially in order to hedge the exposure created by the sale of the puts?
- How much money does the trader need to borrow/lend today in order to make sure that the strategy is self-financing?
- When hedging the puts, should the trader be more worried about gamma or vega risk?

Problem 2 (3 pts). Suppose that the FX Trading desk at Goldman Sachs is analyzing a EUR/USD position for a sovereign wealth fund client. The spot price of the Euro (EUR) is USD 1.15 and the EUR/USD exchange rate has a volatility of 4% per annum. The ECB benchmark rate in Europe is 2.25% per year whereas the Fed Funds rate in the United States is 4.50% per year.

- Calculate the value of a European option to sell EUR 100,000,000 and receive USD 110,000,000 in six months. This represents a notional-weighted strike of 1.10 USD per EUR.
- Use put-call parity to calculate the price of a European option to buy EUR 100,000,000 for USD 110,000,000 in six months. The client is considering this call as an alternative hedging strategy for their upcoming European acquisition.
- Explain why the Black-Scholes formula to buy €1 at time T for a predetermined exchange rate K is given by

$$C = Fe^{-rT} \Phi(d_1) - Ke^{-rT} \Phi(d_2),$$

$$\text{where } d_1 = \frac{\ln(F/K) + \frac{1}{2}\sigma^2 T}{\sigma\sqrt{T}}, \text{ and } d_2 = d_1 - \sigma\sqrt{T}.$$

Note: The table below might come handy to compute $\Phi(-d_1)$ and $\Phi(-d_2)$.

z	$P(Z \leq z)$	z	$P(Z \leq z)$	z	$P(Z \leq z)$	z	$P(Z \leq z)$
-2.00	0.0228	-1.97	0.0244	-1.94	0.0262	-1.91	0.0281
-1.99	0.0233	-1.96	0.0250	-1.93	0.0268	-1.90	0.0287
-1.98	0.0239	-1.95	0.0256	-1.92	0.0274	-1.89	0.0294

Problem 3 (2 pts). Consider a European put option expiring in 6 months and with strike price equal to \$103, written on a stock that currently trades for \$100. Interestingly, the volatility of the stock is zero. The risk-free rate is 5% per year with continuous compounding and the stock pays a dividend yield of 2% per year.

- a. Compute the price of the put option.
- b. Does put-call parity hold if the volatility of the asset returns is equal to zero?

Problem 4 (2 pts). Consider a credit bear spread with strikes K_1 and $K_2 > K_1$ made using European call options written on a non-dividend paying asset and expiring in two months.

- a. In separate diagrams, draw the price, delta and gamma of the bear spread as a function of the stock price.
- b. Determine the sign of the theta if the stock price is equal to K_1 and K_2 , respectively.

Problem 5 (2 pts). Consider a blue-chip tech stock in JP Morgan's equity derivatives portfolio that pays a dividend yield of 2% and has a volatility of returns of 45%. The stock price is \$95 and the risk-free rate is 4.5%.

- a. Compute the price of an asset-or-nothing put that pays 1 share of the stock if the stock price in one month is below \$90. This exotic option was requested by a hedge fund client looking to implement a sophisticated collar strategy.
- b. Compute the price of a cash-or-nothing put that pays \$100 if the stock price in one month is below \$90. The trading desk is considering offering this binary option to complement the client's existing positions.

Problem 6 (2 pts). Calculate the price of a three-month European put option on Bitcoin futures expiring in three months. The three-month futures price is \$89,215, the strike is \$89,000, the risk-free rate is 4.50% and the volatility of the price returns of BTC is 85%.

Problem 7 (2 pts). Determine whether the following statements are true or false and briefly explain why.

- a. A chooser option is very similar to a straddle since at the moment in which you can choose whether you want a call or a put you get pretty much what a straddle pays off.
- b. In order to be able to price a forward-start option in closed-form it is crucial that the option starts at-the-money.

Problem 8 (4 pts). Consider a non-dividend paying stock that trades for \$50. Every 3-months, the stock price can increase or decrease by 10%. The risk-free rate is 5% per year with continuous compounding. Compute the price of the following path-dependent options expiring in 6 months.

- a. A floating lookback call that pays $S_T - S_{min}$ at maturity.
- b. A floating lookback put that pays $S_{max} - S_T$ at maturity.
- c. An average price Asian put option that pays $\max(50 - \bar{S}, 0)$ at maturity.
- d. An average strike Asian call option that pays $\max(S_T - \bar{S}, 0)$ at maturity.

Formula Sheet

In the following, S denotes the stock or spot price of an asset, r is the continuously-compounded risk-free rate expressed per year, δ denotes the dividend yield, T denotes the time-to-maturity of a forward, futures or an option, and K denotes the strike price of an option or the delivery price of a forward contract.

Binomial Pricing

At any node of a binomial tree in which the stock price can move up to $S_u = u \times S$ or down to $S_d = d \times S$, the risk-neutral probability of an up-move is given by

$$q = \frac{Se^{(r-q)\Delta t} - S_d}{S_u - S_d} = \frac{e^{(r-q)\Delta t} - d}{u - d},$$

where Δt denotes the length of each period. To make the tree consistent with the observed volatility of stock returns, we typically choose $u = e^{\sigma\sqrt{\Delta t}}$ and $d = 1/u$.

Impact of Dividends

For European call and put options with strike price K and time-to-expiration T written on a non-dividend paying asset, we have that

$$C - P = Se^{-\delta T} - Ke^{-rT},$$

where C and P denote the call and put prices. Put-call parity implies the following bounds for European call and put options:

$$\begin{aligned}\max(Se^{-\delta T} - Ke^{-rT}, 0) &\leq C \leq S, \\ \max(Ke^{-rT} - Se^{-\delta T}, 0) &\leq P \leq Ke^{-rT}.\end{aligned}$$

Pricing Formulas

In the Black-Scholes model where

$$dS = (r - \delta)Sdt + \sigma SdW,$$

we have the following results for European call and put options. In the formulas,

$$d_1 = \frac{\ln(S/K) + (r - \delta + \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}},$$

and $d_2 = d_1 - \sigma\sqrt{T}$.

Variable	Call	Put
V	$Se^{-\delta T} \Phi(d_1) - Ke^{-rT} \Phi(d_2)$	$Ke^{-rT} \Phi(-d_2) - Se^{-\delta T} \Phi(-d_1)$
Δ	$e^{-\delta T} \Phi(d_1)$	$-e^{-\delta T} \Phi(-d_1)$
Γ	$\frac{e^{-\delta T} \Phi'(d_1)}{S\sigma\sqrt{T}} = \frac{Ke^{-rT} \Phi'(d_2)}{S^2\sigma\sqrt{T}}$	
Θ	$rV - (r - \delta)S\Delta - \frac{1}{2}\sigma^2 S^2 \Gamma$	
$\mathcal{V}$	$Se^{-\delta T} \Phi'(d_1)\sqrt{T} = Ke^{-rT} \Phi'(d_2)\sqrt{T}$	
ρ	$KTe^{-rT} \Phi(d_2)$	$-KTe^{-rT} \Phi(-d_2)$

The Standard Normal Distribution

The following table reports values for $\phi(z) = P(Z \leq z)$, where $Z \sim \mathcal{N}(0, 1)$.

z	$P(Z \leq z)$	z	$P(Z \leq z)$	z	$P(Z \leq z)$	z	$P(Z \leq z)$
-2.37	0.0089	-1.17	0.1210	0.03	0.5120	1.23	0.8907
-2.32	0.0102	-1.12	0.1314	0.08	0.5319	1.28	0.8997
-2.27	0.0116	-1.07	0.1423	0.13	0.5517	1.33	0.9082
-2.22	0.0132	-1.02	0.1539	0.18	0.5714	1.38	0.9162
-2.17	0.0150	-0.97	0.1660	0.23	0.5910	1.43	0.9236
-2.12	0.0170	-0.92	0.1788	0.28	0.6103	1.48	0.9306
-2.07	0.0192	-0.87	0.1922	0.33	0.6293	1.53	0.9370
-2.02	0.0217	-0.82	0.2061	0.38	0.6480	1.58	0.9429
-1.97	0.0244	-0.77	0.2206	0.43	0.6664	1.63	0.9484
-1.92	0.0274	-0.72	0.2358	0.48	0.6844	1.68	0.9535
-1.87	0.0307	-0.67	0.2514	0.53	0.7019	1.73	0.9582
-1.82	0.0344	-0.62	0.2676	0.58	0.7190	1.78	0.9625
-1.77	0.0384	-0.57	0.2843	0.63	0.7357	1.83	0.9664
-1.72	0.0427	-0.52	0.3015	0.68	0.7517	1.88	0.9699
-1.67	0.0475	-0.47	0.3192	0.73	0.7673	1.93	0.9732
-1.62	0.0526	-0.42	0.3372	0.78	0.7823	1.98	0.9761
-1.57	0.0582	-0.37	0.3557	0.83	0.7967	2.03	0.9788
-1.52	0.0643	-0.32	0.3745	0.88	0.8106	2.08	0.9812
-1.47	0.0708	-0.27	0.3936	0.93	0.8238	2.13	0.9834
-1.42	0.0778	-0.22	0.4129	0.98	0.8365	2.18	0.9854
-1.37	0.0853	-0.17	0.4325	1.03	0.8485	2.23	0.9871
-1.32	0.0934	-0.12	0.4522	1.08	0.8599	2.28	0.9887
-1.27	0.1020	-0.07	0.4721	1.13	0.8708	2.33	0.9901
-1.22	0.1112	-0.02	0.4920	1.18	0.8810	2.37	0.9911