Problem Set 4

Instructions: This problem set is due on 11/27 at 11:59 pm CST and is an individual assignment. All problems must be handwritten. Scan your work and submit a PDF file.

Problem 1. The dynamics of X are defined by

$$\mathrm{d}X = \sigma\,\mathrm{d}z.$$

Show that $\mathrm{E}(X_T) = X_0$, and hence that X is a martingale.

Problem 2. Consider two assets whose price processes are given by

$$\frac{\mathrm{d}\mathbf{S}}{\mathbf{S}} = \mu\,\mathrm{d}t + \sigma\,\mathrm{d}\mathbf{z},$$

where $\mathrm{d}\mathbf{z}$ is a vector of three independent Brownian motions z_1, z_2, and z_3. You know that

$$\sigma = \begin{pmatrix} 0.4 & 0.3 & -0.2 \\ 0.5 & -0.2 & -0.1 \end{pmatrix}.$$

a. Compute the instantaneous correlation between the returns of each asset.
b. Find a Brownian motion $z_4 = a_1 z_1 + a_2 z_2 + a_3 z_3$ whose increments are independent from the instantaneous returns of the two assets.

Problem 3. Consider a stochastic discount factor in continuous time given by

$$\frac{\mathrm{d}\Lambda}{\Lambda} = -r\,\mathrm{d}t - \lambda_1\,\mathrm{d}z_1 - \lambda_2\,\mathrm{d}z_2,$$

where z_1 and z_2 are independent Brownian motions. Suppose that you have two non-dividend paying assets with the following dynamics:

$$\frac{\mathrm{d}S_1}{S_1} = \mu_1\,\mathrm{d}t + \sigma_{11}\,\mathrm{d}z_1 + \sigma_{12}\,\mathrm{d}z_2,$$

and

$$\frac{\mathrm{d}S_2}{S_2} = \mu_2\,\mathrm{d}t + \sigma_{21}\,\mathrm{d}z_1 + \sigma_{22}\,\mathrm{d}z_2.$$

Suppose that $r = 0.05$, $\sigma_{11} = 0.5$, $\sigma_{12} = 0.2$, $\sigma_{21} = -0.1$, and $\sigma_{22} = 0.4$. Furthermore, you know that $\lambda_1 = 0.3$ and $\lambda_2 = 0.5$.

a. Compute the instantaneous correlation between the returns of each asset.
b. Compute μ_1 and μ_2.

Problem 4. Suppose that the sales team of a trading desk just sold a European call option contract, i.e., over a 100 shares, to an important client. The contract is written on a non-dividend paying stock that trades for \$210, expires in two years and has a strike price of \$215. The risk-free rate is 6% per year with continuous compounding. A trader of the desk estimate that the volatility of the stock returns is 45% and expected to remain constant for the life of the contract.

a. How many shares of the stock does the trader need to buy/sell initially in order to hedge the exposure created by the sale of the contract?
b. How many risk-free bonds with face value \$215 and expiring in two years does the trader need to buy/sell in order to make sure that the strategy is self-financing?
c. Compute the implicit leverage of call defined as the amount invested in the stock (SC_S) over the price of the call (C).

Problem 5. Consider a European call option expiring in 6 months and with strike price equal to \$42 on a non-dividend paying stock that currently trades for \$40. Interestingly, the volatility of the stock is zero. If the risk-free rate is 6% per year with continuous compounding, what is the price of the option?