Problem Set 3

Instructions: This problem set is due on 11/13 at 11:59 pm CST and is an individual assignment. All problems must be handwritten. Scan your work and submit a PDF file.

Problem 1. Find the derivative of $f(x)$ with respect to x.

1. $f(x) = 10x$
2. $f(x) = 4x - 5$
3. $f(x) = 100x - 3x^2$
4. $f(x) = -10x + 6x^2 - \frac{2}{3}x^3$
5. $f(x) = 5x^{\frac{1}{3}}$
6. $f(x) = \frac{1}{x^{\frac{1}{5}}}$
7. $f(x) = e^{-\frac{1}{2}x}$
8. $f(x) = \ln\left(\frac{1}{x}\right)$

Problem 2. Compute the following integrals.

1. $\int_0^1 e^{-0.1t}\,\mathrm{d}t$
2. $\int_0^1 3x^5\,\mathrm{d}x$
3. $\int_0^1 \frac{1}{x+5}\,\mathrm{d}x$

Problem 3. Consider an asset that pays a continuous cash flow $ce^{gt}\,\mathrm{d}t$ from time 0 up to time T. The interest rate is r with continuous compounding

a. Compute the value of the asset at time 0.
b. Compute the value of the asset at time $t < T$.
c. What should be the value of the asset at time T?

Problem 4. Let S be the price of TESLA stock that follows a geometric Brownian motion such that

$$\mathrm{d}S = \mu S\,\mathrm{d}t + \sigma S\,\mathrm{d}W.$$

Your sales team would like to launch a new product called TESLA Quadro that tracks the price of TESLA to the power 4. In other words, the value of this instrument is given by $Y = S^4$. What is the process followed by Y?

Problem 5. Suppose that the stock price follows a geometric Brownian motion (GBM) with drift μ and instantaneous volatility σ. Show that $Y = Se^{-\mu t}$ also follows a GBM and determine the drift and volatility as a function of μ and σ.

Problem 6. Suppose that the stock price follows a geometric Brownian motion (GBM) with drift r and instantaneous volatility σ, where r is the risk-free rate. Consider the futures price of S at time t and expiring at T, given by $F = Se^{r(T-t)}$. Show that F has zero drift and hence is a martingale.