Problem Set 2

Instructions: This problem set is due on 9/20 at 11:59 pm CST and is an individual assignment. All problems must be handwritten. Scan your work and submit a PDF file.

Problem 1. Suppose that X is a normally distributed random variable with mean $\mu = 12$ and standard deviation $\sigma = 20$.

a) What is the probability that $X \leq 0$?
b) What is the probability that $X \leq -4$?
c) What is the probability that $X > 8$?
d) What is the probability that $4 < X \leq 10$?

Problem 2. Suppose that X is a normally distributed random variable with mean $\mu = 10$ and standard deviation $\sigma = 20$. Compute the 90%, 95%, and 99% confidence interval for X.

Problem 3. Suppose that $X = \ln(Y)$ is a normally distributed random variable with mean $\mu = 3.9$ and standard deviation $\sigma = 15$.

a) What is the probability that $Y \leq 6$?
b) What is the probability that $Y > 4$?
c) What is the probability that $3 < Y \leq 12$?
d) What is the probability that $Y \leq 0$?

Problem 4. Suppose that X is a normally distributed variable with mean $\mu = 3.70$ and standard deviation $\sigma = 0.80$. If $Y = e^X$, what is the probability that Y is greater than 45?

Problem 5. Let $Y = e^{\mu + \sigma Z}$ where $\mu = 1$, $\sigma = 2$ and $Z \sim \mathcal{N}(0, 1)$. Compute:

a) $E(Y)$
b) $SD(Y) = \sqrt{E(Y^2) - E(Y)^2}$
c) $E(Y^{0.3})$
d) $E(Y^{-1})$

Problem 6. Consider a stock whose price at time T is given by S_T such that,

$$\ln(S_T) \sim \mathcal{N}\left(\ln(S_0) + (\mu - 0.5\sigma^2)T, \sigma^2 T\right).$$

The expected return is 12% per year and the volatility is 35% per year. The current spot price is $100.

a) Compute the expected price in 2 years from now.
b) Compute the mean and standard deviation of the log-spot price in 2 years from now.
c) Compute the probability that the spot price is less than $100 in 2 years from now.
d) Compute the probability that the spot price is greater than $120 in 2 years from now.

Problem 7. Consider a stock whose price at time T is given by S_T such that,

$$\ln(S_T) \sim \mathcal{N}\left(\ln(S_0) + (\mu - 0.5\sigma^2)T, \sigma^2 T\right).$$

The expected return is 18% per year and the volatility is 32% per year. The current spot price is $60.

a) Compute the expected price 9 months from now.
b) Compute the mean and standard deviation of the log-spot price 9 months from now.
c) Compute the 95% confidence interval of $\ln(S_T)$ 9-months from now, and report the corresponding values for S_T.