Problem Set 1

Instructions: This problem set is due on 10/30 at 11:59 pm CST and is an individual assignment. All problems must be handwritten. Scan your work and submit a PDF file.

Problem 1. Suppose the evolution of y_t given by:

$$y_t = 0.1 + 0.90 y_{t-1}.$$

a. Compute y_{10} if you know that $y_1 = 3$.
b. Sketch a graph of y_t as a function of t, specifying the value of $\bar{y} = \lim_{t \to \infty} y_t$.

Problem 2 (Exponential Growth)**.** Let $y_t = e^{x_t}$, and consider the process

$$x_{t+1} = a + x_t, \tag{1}$$

where $a > 1$.

a. Compute y_t as function of y_0, a and t.
b. Sketch a graph of y_t as function of t.

Problem 3 (Exponential Growth with Capacity)**.** Let $y_t = e^{x_t}$, and consider the process

$$x_{t+1} = x_t \left(1 + r\left(1 - \frac{x_t}{K}\right)\right), \tag{2}$$

given $0 < x_0 < K$ and $0 < r < 1$.

a. Explain what should happen to the growth rate of x_t as $t \to \infty$.
b. Assuming that $\bar{x} = \lim_{t \to \infty} x_t$ exists, compute $\bar{x}$.
c. Sketch a graph of y_t as a function of t, specifying the value of $\lim_{t \to \infty} y_t$. How this plot differs from the one in Exercise 2?

Problem 4 (Applying the DDM)**.** ACME last year paid a dividend of $3.40 per share. This dividend is expected to grow at 20% per year for the next five years, after which it is expected to grow at 3% in perpetuity.

a. What is the stock's value if your required rate of return is 10%?
b. Would the price change if you expected to hold the share for only three years?

Problem 5 (Fibonacci Numbers)**.** The Fibonacci numbers are defined by the recurrence relation

$$F_0 = 0, \quad F_1 = 1, \tag{3}$$

and

$$F_n = F_{n-1} + F_{n-2} \tag{4}$$

for $n > 1$.

a. Compute the first 10 Fibonacci numbers using (3) and (4).
b. Does the Fibonacci sequence F_n converges to a finite number as $n \to \infty$?
c. Write (4) as a system of two difference equations and indicate the initial conditions of each variable.

Problem 6. Consider the system

$$x_t = 0.95x_{t-1} + w_t,$$

where $x_{-1} = 0$, and

$$w_t = \begin{cases} 1 & \text{if } t = 0, \\ 0 & \text{if } t > 0. \end{cases}$$

a. Sketch a graph of x_t as a function of t, specifying the value of $\bar{x} = \lim_{t\to\infty} x_t$.
b. Determine $t \in \mathbb{N}$ such that x_t is the closest to 0.5.

Problem 7. A bank account gives you an interest rate of r per period. Denote by x_t the balance of your bank account at time t, and assume that at the end of each period you withdraw the interest paid by the bank account, leaving the principal intact.

a. Write down the difference equation relating x_t to x_{t-1}.
b. Using the previous result, derive the present value of a perpetuity paying a constant cash flow c at the end of each period indefinitely.