

## Problem Set 5

**Instructions:** This problem set is due on 10/3 at 11:59 pm CST and is an individual assignment. All problems must be handwritten. Scan your work and submit a PDF file.

### Bond Pricing

**Problem 1.** Your company aims to raise \$10 million by issuing 20-year zero-coupon bonds. With a yield to maturity of 6% per year, compounded annually, what should be the total face value of the bonds to achieve this goal?

**Problem 2.** Suppose a seven-year, \$1,000 bond with an 8% coupon rate and semiannual coupons is trading with a yield to maturity of 6.75%.

- Is this bond currently trading at a discount, at par, or at a premium? Explain.
- If the YTM of the bond suddenly rises to 7% (APR with semiannual compounding), what price will the bond trade for?

**Problem 3.** Suppose that General Motors Acceptance Corporation issued a bond with 10 years until maturity, a face value of \$1,000, and a coupon rate of 7% (annual payments). The yield to maturity on this bond when it was issued was 6%.

- What was the price of this bond when it was issued?
- Assuming the yield to maturity remains constant, what is the price of the bond immediately before it makes its first coupon payment?

**Problem 4.** Your company currently has \$1,000 par, 6% coupon bonds with 10 years to maturity and a yield-to-maturity of 5% per year with semi-annual compounding. If you want to issue new 10-year coupon bonds at par, what coupon rate do you need to set? Assume that for both bonds, the next coupon payment is due in exactly six months.

**Problem 5.** Derive the probability distribution of the 1-year HPR on a 30-year U.S. Treasury bond with an 8% coupon if it is currently selling at par and the probability distribution of its YTM a year from now is as follows:

Economy	Probability	YTM	Price	HPR
Boom	0.2	11%		
Normal	0.5	8%		
Recession	0.3	7%		

Assume that the entire 8% coupon is paid at the end of the year rather than every 6 months over a principal of \$100.

## Forward Rates

**Problem 6.** Below is a list of prices for \$1,000 par zero-coupon bonds of various maturities.

Bond	Maturity (years)	Price (\$)
Z(1)	1	930
Z(2)	2	850
Z(3)	3	770
Z(4)	4	700

- Compute the zero-coupon rates for years 1, 2, 3 and 4. Express the rates per year with annual compounding.
- Consider an 8% coupon \$1,000 par bond (denoted by B) paying annual coupons and expiring in 4 years. Compute the no-arbitrage price of the bond and its yield-to-maturity.
- If the expectations hypothesis holds, what is your forecast for the 3-year interest rate (per year compounded annually) expected next year?

- d. If bond B was trading today for \$985, is there an arbitrage opportunity that can be exploited? If so, explain how an investor would exploit such a strategy, i.e. indicate which securities the investor would buy or sell, as well as the quantities.

**Problem 7.** Below is a list of zero-coupon rates expressed per year with annual compounding for various maturities:

Maturity (years)	1	2	3
Zero Rate	10%	9%	8%

- Compute the prizes of zero-coupon bonds (\$1,000 face value) with maturities 1, 2 and 3 years.
- Compute the current forward rates (per year with annual compounding) from years 1 to 2, from 2 to 3 and from 1 to 3.
- Consider an 8.5% coupon (\$1,000 face value) bond paying annual coupons and expiring in 3 years. Compute the no-arbitrage price of the bond.
- If at the end of the first year the yield curve flattens out at 10% for all maturities, what will be the 1-year holding-period return (per year with continuous compounding) on the coupon bond?
- If the coupon bond described in c. was instead trading today for \$1,000, is there an arbitrage opportunity? If so, explain how an investor would exploit such a strategy, i.e. indicate which securities the investor would buy or sell, as well as the quantities.

## Interest Rate Risk

**Problem 8.** An insurance company must make payments to a customer of \$10 million in 5 years and \$25 million in 30 years. The yield curve is flat at 8% per year with annual compounding.

- What is the present value and duration of its obligation?
- If it wants to fully fund and immunize its obligation to this customer with a single issue of a zero-coupon bond, what maturity bond must it purchase?

- c. Suppose you buy a zero-coupon bond with value and duration equal to your obligation, and that rates immediately increase to 9%. What happens to your net position, that is, to the difference between the value of the bond and that of your insurance obligation?

**Problem 9.** An insurance company must make payments to a customer of \$20 million in 10 years and \$40 million in 20 years. The yield curve is flat at 6% per year with annual compounding.

- a. What is the present value and duration of its obligation?
- b. If it wants to fully fund and immunize its obligation to this customer with a single issue of a zero-coupon bond, what zero-coupon bond must it purchase? Specify the maturity and face value of the bond.
- c. Suppose that instead of using a single zero-coupon bond as in part b, the insurance company plans to use the following bonds to immunize its exposure to interest rate risk:

Bond	Duration (years)
$B_5$	5
$B_{30}$	30

Determine the total amount to invest in each bond that the insurance company must buy to immunize its obligation.