Investment Theory | Prof. Lorenzo Naranjo
FIN 5320 | Fall 2025

Problem Set 3

Instructions: This problem set is due on 9/19 at 11:59 pm CST and is an individual assignment. All problems must be handwritten. Scan your work and submit a PDF file.

Problem 1. A pension fund manager is considering three mutual funds. The first is a stock fund, the second is a long-term government and corporate bond fund, and the third is a T-bill money market fund that yields a rate of 8%. The probability distribution of the risky funds is as follows:

Fund	Expected Return	Standard Deviation
Stock fund (S)	20%	30%
Bond fund (B)	12%	15%

The correlation between the fund returns is 0.10.

a. What are the investment proportions in the minimum-variance portfolio of the two risky funds.

b. What is the expected value and standard deviation of the minimum variance portfolio rate of return?

c. Compute the expected return and standard deviation of the following portfolios.

Proportion in S	Proportion in B	Expected Return	Standard Deviation
25%	75%		
50%	50%		
75%	25%		

Problem 2. Suppose that there are many stocks in the security market and that the characteristics of stocks A and B are given as follows:

Stock	Expected Return	Standard Deviation
A	10%	5%
B	15%	10%

The correlation between the stock returns is -1. Suppose that it is possible to invest and borrow at the risk-free rate, r_f. What must be the value of the risk-free rate?

Problem 3. Abigail Grace has a $1,400,000 fully diversified portfolio. She subsequently inherits ABC Company common stock worth $200,000. Her financial adviser provided her with the following forecast information:

Asset	Expected Return	Standard Deviation
Original Portfolio	9.2%	8%
ABC Company	16.0%	16%

The correlation coefficient of ABC stock returns with the original portfolio returns is 0.40.

a. The inheritance changes Grace's overall portfolio and she is deciding whether to keep the ABC stock. Assuming Grace keeps the ABC stock, calculate the expected return and standard deviation of her new portfolio, which includes the ABC stock.
b. If Grace sells the ABC stock, she will invest the proceeds in risk-free government securities yielding 6%. Assuming Grace sells the ABC stock and replaces it with the government securities, calculate the expected return and standard deviation of her new portfolio, which includes the government securities.
c. Based on conversations with a friend, Grace is considering selling the $200,000 of ABC stock and acquiring $200,000 of XYZ Company common stock instead. XYZ stock has the same expected return and standard deviation as ABC stock. Her friend comments, "It doesn't matter whether you keep all of the ABC stock or replace it with $200,000 of XYZ stock." State whether her friend's comment is correct or incorrect. Justify your response briefly.

Problem 4. Consider an economy spanned by many risky assets and a risk-free asset that yields 5%. The tangency portfolio (Q) has an expected return of 20% and a standard deviation of 30%. Furthermore, you have information about the following funds:

Fund	Expected Return	Standard Deviation
A	15%	25%
B	8%	15%

The correlation between A and B is 0.24.

a. In a (σ, μ) diagram, draw the capital allocation line (CAL) of Q, and plot funds A and B. Which portfolios are efficient? Why?
b. Peter wants to invest in an efficient portfolio (i.e. maximum Sharpe ratio) that offers an expected return of 30%. What should he do? Specify in which assets he should invest, and the standard deviation of such portfolio.
c. Christine, on the other hand, for regulatory reasons can only invest in funds A and B (and not the risk-free asset). She's aiming for an expected return of 10%. What would you recommend to her? Please clearly indicate the composition and the standard deviation of such portfolio. Is her portfolio efficient?

Problem 5. Two risky assets A and B have the following characteristics:

- $\mathrm{E}(R_A) = 0.10, \sigma_A = 0.10$
- $\mathrm{E}(R_B) = 0.15, \sigma_B = 0.25$
- $\rho_{AB} = +1$

a. Suppose that you aim for an expected return of 5%, what should be the composition of your portfolio? What is the risk of that portfolio? Explain how this strategy can be possible.
b. Draw the investment opportunity set and clearly identify the minimum variance portfolio. Determine its composition and its characteristics.
c. Suppose that in addition to A and B, there is a risk-free asset offering a return of 4%. Is there an arbitrage opportunity? If so, clearly explain how an investor could arbitrage this opportunity (what he buys and what he sells).

Problem 6. Consider an economy spanned by N risky assets and a risk-free asset. The tangency portfolio (Q) has an expected return of 20% and a standard deviation of 30%. You also know that Peter chooses to optimally invest in portfolio (P_1) with an expected return of 12% and a standard deviation of 15%.

a. Compute the risk-free rate r_f of this economy.
b. What is the composition of the portfolio owned by Peter?
c. Michael wants to invest optimally in a portfolio (P_2) that has a standard deviation of 40%. What would you recommend to him? Please clearly indicate the composition of his portfolio, and the expected return that he will be able to achieve.

Problem 7. Karen Kay, a portfolio manager at Collins Asset Management, is using the capital asset pricing model for making recommendations to her clients. Her research department has developed the information shown in the following exhibit.

	Forecast Return	Standard Deviation	Beta
Stock X	15%	37%	0.7
Stock Y	18%	26%	1.6
Market Index (M)	15%	16%	1.0
Risk-Free Rate (F)	5%		

a. Calculate the CAPM expected return and alpha for each stock. Use the forecast return of the market as a proxy for the expected market return.
b. Calculate the Sharpe ratio for each stock and the market using the forecast return as a proxy for expected return.
c. Identify and justify which stock (X or Y) would be more appropriate for an investor who wants to

   i. add this stock to a well-diversified equity portfolio.
   ii. hold this stock as a single-stock portfolio.