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Sequential Trading

Introduction

The bid–ask spread is the difference between the price at which a dealer is willing to buy a security (the bid) and the price at which the dealer is willing to sell it (the ask). Dealers or quasi-dealers provide liquidity by posting two-sided quotes and seek to capture the spread by buying at the bid and selling at the ask. Occasional traders, by contrast, often submit limit orders as alternatives to market orders and typically exit the market upon execution. The size of the spread compensates liquidity providers for inventory risk, order-processing costs, and, crucially, adverse selection when some traders possess superior information.

The pricing of liquidity depends on whether order flow is informative about the security's fundamental value. If some traders possess superior knowledge, the stream of incoming buy and sell orders becomes correlated with the true value, and a dealer who trades against such informed flow suffers losses. When order flow is uninformative—driven by portfolio rebalancing or liquidity needs—the dealer can choose quotes to balance arrivals of buyers and sellers, earning the spread on average. When order flow is partially informative, the dealer faces adverse selection: informed traders buy before price increases and sell before price decreases. To remain viable, the dealer sets a spread wide enough that expected profits from uninformed traders cover expected losses to informed traders.

Sequential Trading Framework

In a sequential trading framework, orders arrive one at a time, and each trade conveys information through its direction. The dealer cannot observe whether the counterparty is informed or uninformed, so quotes are updated after each trade as the dealer revises beliefs about the asset's true value. Trading costs, primarily the bid–ask spread, erode returns for many investors. This spread arises endogenously from competition among dealers who must break even in expectation while facing informed order flow.

Consider a single asset with a terminal value $V \in \{V_d, V_u\}$ where $V_d < V_u$. The prior probabilities $\mathrm{P}(V = V_u) = p$ and $\mathrm{P}(V = V_d) = 1 - p$ represent beliefs about the asset's value. The expected value of V is

$$\mathrm{E}(V) = pV_u + (1 - p)V_d.$$

Each arriving trader is either informed or uninformed. Let $\alpha = \mathrm{P}(\text{informed})$ and $1 - \alpha = \mathrm{P}(\text{uninformed})$. Uninformed traders execute buy or sell orders with equal probability $1/2$, regardless of V. Informed traders, knowing V, buy when $V = V_u$ and sell when $V = V_d$. The dealer, unable to distinguish between informed and uninformed traders, sets quotes before observing the next order. To do so, the dealer analyzes the implications of the next order being a sell or a buy.

The dealer buys at the bid from a trader who wants to sell and sells at the ask to a trader who wants to buy. Denoting $\mathrm{E}(V \mid \text{Sell})$ as the expectation of V conditional on a sell order and $\mathrm{E}(V \mid \text{Buy})$ as the expectation of V conditional on a buy order, the dealer sets $\text{Bid} = \mathrm{E}(V \mid \text{Sell})$ and $\text{Ask} = \mathrm{E}(V \mid \text{Buy})$.

Computing the Bid Price

Step 1

Conditional on $V = V_u$, the sell probability is

$$\mathrm{P}(\text{Sell} \mid V = V_u) = 0\alpha + \frac{1}{2}(1 - \alpha),$$

since no informed trader sells (they know the value is high) and half of the noise traders sell (they always do). Similarly,

$$\mathrm{P}(\text{Sell} \mid V = V_d) = 1\alpha + \frac{1}{2}(1 - \alpha),$$

since now all informed traders sell (they know the value is low) and half of the noise traders sell. Finally, by the law of total probability,

$$\begin{aligned}\mathrm{P(Sell)} &= \mathrm{P(Sell} \mid V = V_u)\,\mathrm{P}(V = V_u) + \mathrm{P(Sell} \mid V = V_d)\,\mathrm{P}(V = V_d) \\ &= \frac{1}{2}(1-\alpha)p + \left(\alpha + \frac{1}{2}(1-\alpha)\right)(1-p) \\ &= \frac{1}{2}(1-\alpha) + (1-p)\alpha.\end{aligned}$$

Note that in P(Sell) the first term corresponds to the proportion of noise traders that always sell (half of them) and the second term corresponds to the proportion of informed traders that sell only when the price is low.

Step 2

We now apply Bayes theorem to find

$$\mathrm{P}(V = V_u \mid \mathrm{Sell}) = \frac{\mathrm{P(Sell} \mid V = V_u)\,\mathrm{P}(V = V_u)}{\mathrm{P(Sell)}} = \frac{\frac{1}{2}(1-\alpha)p}{\frac{1}{2}(1-\alpha) + (1-p)\alpha},$$

and

$$\mathrm{P}(V = V_d \mid \mathrm{Sell}) = 1 - \mathrm{P}(V = V_u \mid \mathrm{Sell}).$$

Note that $\mathrm{P}(V = V_u \mid \mathrm{Sell}) < p$ and $\mathrm{P}(V = V_d \mid \mathrm{Sell}) > 1 - p$. Indeed, a sell order should decrease the posterior probability that the value is high and increase the posterior probability that the value is low. Thus,

$$\mathrm{Bid} = \mathrm{E}(V \mid \mathrm{Sell}) = V_u\,\mathrm{P}(V = V_u \mid \mathrm{Sell}) + V_d\,\mathrm{P}(V = V_d \mid \mathrm{Sell}) < \mathrm{E}(V).$$

Computing the Offer Price

As before, we compute

$$\mathrm{P(Buy} \mid V = V_u) = 1\alpha + \frac{1}{2}(1-\alpha),$$

since now all informed traders buy (they know the value is high) and half the noise traders buy. Similarly,

$$\mathrm{P}(\mathrm{Buy} \mid V = V_d) = 0\alpha + \frac{1}{2}(1-\alpha),$$

since no informed trader buys (they know the value is low) and half of the noise traders sell (they always do). Also,

$$\mathrm{P}(\mathrm{Buy}) = 1 - \mathrm{P}(\mathrm{Sell}) = \frac{1}{2}(1-\alpha) + p\alpha.$$

Using Bayes theorem again we get

$$\mathrm{P}(V = V_d \mid \mathrm{Buy}) = \frac{\frac{1}{2}(1-\alpha)(1-p)}{\frac{1}{2}(1-\alpha) + p\alpha} < 1 - p,$$

and

$$\mathrm{P}(V = V_u \mid \mathrm{Buy}) = 1 - \mathrm{P}(V = V_d \mid \mathrm{Buy}) > p.$$

Thus,

$$\mathrm{Ask} = \mathrm{E}(V \mid \mathrm{Buy}) = V_u\, \mathrm{P}(V = V_u \mid \mathrm{Buy}) + V_d\, \mathrm{P}(V = V_d \mid \mathrm{Buy}) > \mathrm{E}(V).$$

Analysis

The previous results show that an incoming trade moves the bid and offer price regardless of whether it is initiated by a noise or an informed trader. Indeed, the market maker does not know who is behind the trade, and therefore uses the trade as an imperfect signal to determine the true value of the security. Before a trade arrives, the market maker determines the bid and offer such that Bid $<$ E(V) $<$ Ask. How wide is the bid-ask spread depends of course of the proportion of informed traders in the market.

Each trade direction changes posterior beliefs and thus moves quotes in the direction of the order. A sequence of sells pushes both bid and ask toward the low value; a sequence of buys pushes them toward the high value. As trading accumulates, uncertainty declines and the spread narrows; prices converge toward the true value. Because the dealer cannot distinguish informed from uninformed orders, even uninformed trades move prices, generating temporary mispricing that persists until new public information arrives or the continued absence of news convinces the market otherwise.

Note that both traders are important for the market to function well. If there are no informed traders, trading costs are zero but prices are inefficient since nobody is trading on information. If there are too many informed traders prices quickly incorporate information but trading is expensive as the bid-ask spread is wider. Both types of traders contribute to the efficiency and liquidity of the market.

Examples

Example 1 (Symmetric Prior)**.** Assume that the true value of the stock is either \$100 or \$150 with equal probabilities. Furthermore, assume that 20% of the market is composed of informed traders so 80% of the market are noise traders. Thus, we have $V_u = 150$, $V_d = 100$, $p = \frac{1}{2}$ and $\alpha = 0.2$. Bayes' rule implies $\mathrm{P}(V = 150 \mid \text{Sell}) = 0.4$ and $\mathrm{P}(V = 100 \mid \text{Buy}) = 0.6$. Hence

$$\text{Bid} = 0.6 \times 100 + 0.4 \times 150 = \$120,$$

and

$$\text{Ask} = 0.4 \times 100 + 0.6 \times 150 = \$130.$$

Thus, the bid-ask spread equals \$10. □

Example 2 (Asymmetric Prior)**.** Using the numbers of Example 1, assume now that $p = 0.4$. Then

$$\mathrm{P}(V = 150 \mid \text{Sell}) = \frac{\frac{1}{2}(1-0.2)0.4}{\frac{1}{2}(1-0.2) + 2(1-0.4)0.2} = \frac{4}{13},$$

and $\mathrm{P}(V = 100 \mid \text{Sell}) = 1 - \frac{4}{13} = \frac{9}{13}$. Hence

$$\text{Bid} = 100 \times \frac{9}{13} + 150 \times \frac{4}{13} \approx \$115.3846.$$

Similarly,

$$\text{P}(V = 100 \mid \text{Buy}) = \frac{\frac{1}{2}(1-0.2)(1-0.4)}{\frac{1}{2}(1-0.2)+0.40.2} = \frac{1}{2},$$

so that

$$\text{Ask} = 100 \times 0.5 + 150 \times 0.5 = \$125.$$

Relative to the symmetric case, the mid-quote shifts downward because the prior probability of the low state is greater, and the spread is asymmetric around the unconditional expectation. □