Topics in Quantitative Finance | Prof. Lorenzo Naranjo
FIN 5018 | Fall 2025

Problem Set 5

Instructions: This problem set is due on 10/13 at 11:59 pm CST and is an individual assignment. All problems must be handwritten. Scan your work and submit a PDF file.

Problem 1. In this problem all parameters are constant, and the time interval is $[0, T]$. A non-dividend paying stock S follows a GBM

$$\frac{dS}{S} = \mu_S dt + \sigma_S dB.$$

The stochastic discount factor is such that

$$\frac{d\Lambda}{\Lambda} = -r dt - \lambda dB_\Lambda,$$

where $dB dB_\Lambda = \rho_{S,\Lambda} dt$. As always, the money-market account β satisfies $d\beta = r\beta dt$.

a. Compute

$$dB^* = dB - \frac{d\mathcal{E}}{\mathcal{E}} dB,$$

where $\mathcal{E} = \Lambda\beta$.

b. Determine the dynamics of S under the risk-neutral measure P^* defined as

$$\frac{d\,\mathrm{P}^*}{d\,\mathrm{P}} = \mathcal{E}_T.$$

c. Compute

$$dB^S = dB - \frac{d\mathcal{E}^S}{\mathcal{E}^S} dB,$$

where $\mathcal{E}^S = \Lambda S$.

d. Determine the dynamics of S under the alternative measure P^S defined as

$$\frac{d\,\mathrm{P}^S}{d\,\mathrm{P}} = \mathcal{E}^S_T.$$

e. In class we saw that the price of a European call option written on S with strike price K and expiring at T is

$$C = S\,\mathsf{P}^S(S_T > K) - Ke^{-rT}\,\mathsf{P}^*(S_T > K).$$

In the formula, explain why the risk-adjusted probability of the first term is different from the risk-adjusted probability of the second term.

Problem 2. In the model of Schwartz and Smith (2000), the spot price of a commodity is modelled as

$$S = e^{x+y},$$

where

$$dx = \mu dt + \sigma_x dB_x,$$
$$dy = -\kappa y dt + \sigma_y dB_y,$$

and $dB_x dB_y = \rho_{x,y} dt$. In class we saw that we can solve for x_T and y_T as

$$x_T = x_0 + \mu T + \sigma_x \int_0^T dB_{xt},$$
$$y_T = y_0 e^{-\kappa T} + \sigma_y e^{-\kappa T} \int_0^T e^{\kappa t} dB_{yt},$$

a. Explain why x captures permanent shocks to S.
b. Explain the mechanism that makes shocks to y to be mean-reverting.
c. Explain intuitively why x_T and y_T are jointly normally distributed.

Problem 3. Suppose that a non-dividend paying stock S follows a geometric Brownian motion under the risk-neutral measure such that

$$\frac{dS}{S} = rdt + \sigma dB^*,$$

where r and σ are constants. Compute the futures price of S expiring at time T as $F(T) = \mathsf{E}^*(S_T)$.