Topics in Quantitative Finance | Prof. Lorenzo Naranjo
FIN 5018 | Fall 2025

Problem Set 3

Instructions: This problem set is due on 9/19 at 11:59 pm CST and is an individual assignment. All problems must be handwritten. Scan your work and submit a PDF file.

Short Questions

Problem 1. We saw in class that the stochastic discount factor can be derived from aggregate consumption as:

$$m_{t+1} = \beta \frac{u'(c_{t+1})}{u'(c_t)}.$$

Is the stochastic discount factor going to be high or low when consumption at time $t+1$ is low? Why?

Problem 2. Briefly explain the relation between the principle of no-arbitrage (PNA) and the law of one price (LOOP). What are the implications of each assumption regarding stochastic discount factors?

Problem 3. Consider a risky asset with total returns given by R. If the covariance of the asset's returns with the stochastic discount factor is high, should the expected return of the risky asset be high or low? what about its current price?

Problem 4. Consider a risky asset with total returns given by R and two stochastic discount factors m_1 and m_2 that price the asset correctly. Compute $\mathrm{E}((m_1 - m_2)R)$.

Problem 5. Consider two risky assets with total returns given by R_i and R_j, respectively. If m is a stochastic discount factor that prices both assets correctly, compute $\mathrm{E}((R_i - R_j)m)$. How do we typically call the difference between the two assets' returns?

Problem 6. Suppose that uncertainty is modeled by a finite probability space with three possible states of nature. If there are three traded risky assets such that no combination of two of them can create the third, is it possible to find two distinct stochastic discount factors?

Practicing Projections

Problem 7. Suppose that you have the following information about a risky asset x and a stochastic discount factor m.

Probability	x	m
0.2	20	0.7
0.4	14	0.8
0.3	6	1.2
0.1	4	1.6

a. Compute the risk-free rate implied by the SDF.
b. Compute the price of x and its expected return.
c. Compute the projection of the SDF onto the subspace generated by a constant and x.
d. Verify that the projection computed in (c) is a valid SDF that gives the same price for x and the risk-free rate.