Problem Set 7

Instructions: This problem set is due on 4/21 at 11:59 pm CST and is an individual assignment. All problems must be handwritten. Scan your work and submit a PDF file. For all probability computations, use the table at the end of this assignment.

Note: All interest rates and dividend yields are expressed per year with continuous compounding.

Problems

Problem 1. Suppose that the spot price of the Canadian dollar (CAD) is USD 0.72 and that the CAD/USD exchange rate has a volatility of 5% per annum. The risk-free interest rates in Canada and the United States are 2.75% and 4.50% per annum, respectively. Calculate the value of a European option to buy CAD 10,000,000 for USD 7,500,000 in six months. Use put-call parity to calculate the price of a European option to sell CAD 10,000,000 for USD 7,500,000 in six months.

Problem 2. A futures price is currently 70, it's volatility is 30% per year, and the risk-free rate is 5% per year. What is the value of a six-month European call on the futures with a strike price of 72.5?

Problem 3. Calculate the price of a three-month European call option on the spot value of BTC. The three-month futures price is $86,225, the strike is $86,000, the risk-free rate is 4.50% and the volatility of the price returns of BTC is 80%.

Problem 4. Explain intuitively why the risk-neutral drift of the futures price returns is equal to zero.

Problem 5. What does it mean if the theta of an option's position is -0.2 when time is measured in years? If a trader feels that the stock price can change quickly in any direction, what type of option position is appropriate? How the negative theta affects this option position if the trader is wrong?

Problem 6. What are the risks for a trader if the gamma of a position is large and positive and the delta is zero?

Problem 7. Consider a bull spread made using European call options written on a non-dividend paying asset expiring in a month. In separate diagrams, draw the price, delta and gamma of the bull spread as a function of the stock price.

Problem 8. A financial institution has the following portfolio of over-the-counter options on the British pound.

Type	Position	Delta of option	Gamma of option	Vega of option
Call	-2,000	0.5	0.008	0.40
Call	800	0.8	0.020	0.05
Put	-2,000	-0.4	0.012	0.12
Put	1,500	-0.6	0.007	0.42

A traded option is available with a delta of -0.4, a gamma of 0.011, and a vega of 0.08. Please answer the following:

a. What position in the traded option and the euro, would make the portfolio both gamma neutral and delta neutral?
b. What position in the traded option and the euro, would make the portfolio both vega neutral and delta neutral?

Problem 9. Which of the following options strategies have **negative theta**? Assume that all options are European and written on an asset that does not pay dividends and that the risk-free rate is positive. Briefly explain why.

a. A short straddle with a strike price equal to the current stock price.
b. A long strangle with strike prices $K_1 < K_2$ in which the current stock price is between K_1 and K_2.
c. A bull spread with strike prices $K_1 < K_2$ in which the current stock price is more than K_2.
d. A bear spread with strike prices $K_1 < K_2$ in which the current stock price is more than K_2.

The Standard Normal Distribution

The following table reports values for $\phi(z) = P(Z \leq z)$, where $Z \sim \mathcal{N}(0, 1)$.

z	P(Z ≤ z)	z	P(Z ≤ z)	z	P(Z ≤ z)	z	P(Z ≤ z)
-2.37	0.0089	-1.17	0.1210	0.03	0.5120	1.23	0.8907
-2.32	0.0102	-1.12	0.1314	0.08	0.5319	1.28	0.8997
-2.27	0.0116	-1.07	0.1423	0.13	0.5517	1.33	0.9082
-2.22	0.0132	-1.02	0.1539	0.18	0.5714	1.38	0.9162
-2.17	0.0150	-0.97	0.1660	0.23	0.5910	1.43	0.9236
-2.12	0.0170	-0.92	0.1788	0.28	0.6103	1.48	0.9306
-2.07	0.0192	-0.87	0.1922	0.33	0.6293	1.53	0.9370
-2.02	0.0217	-0.82	0.2061	0.38	0.6480	1.58	0.9429
-1.97	0.0244	-0.77	0.2206	0.43	0.6664	1.63	0.9484
-1.92	0.0274	-0.72	0.2358	0.48	0.6844	1.68	0.9535
-1.87	0.0307	-0.67	0.2514	0.53	0.7019	1.73	0.9582
-1.82	0.0344	-0.62	0.2676	0.58	0.7190	1.78	0.9625
-1.77	0.0384	-0.57	0.2843	0.63	0.7357	1.83	0.9664
-1.72	0.0427	-0.52	0.3015	0.68	0.7517	1.88	0.9699
-1.67	0.0475	-0.47	0.3192	0.73	0.7673	1.93	0.9732
-1.62	0.0526	-0.42	0.3372	0.78	0.7823	1.98	0.9761
-1.57	0.0582	-0.37	0.3557	0.83	0.7967	2.03	0.9788
-1.52	0.0643	-0.32	0.3745	0.88	0.8106	2.08	0.9812
-1.47	0.0708	-0.27	0.3936	0.93	0.8238	2.13	0.9834
-1.42	0.0778	-0.22	0.4129	0.98	0.8365	2.18	0.9854
-1.37	0.0853	-0.17	0.4325	1.03	0.8485	2.23	0.9871
-1.32	0.0934	-0.12	0.4522	1.08	0.8599	2.28	0.9887
-1.27	0.1020	-0.07	0.4721	1.13	0.8708	2.33	0.9901
-1.22	0.1112	-0.02	0.4920	1.18	0.8810	2.37	0.9911