Problem Set 5

Instructions: This problem set is due on 3/28 at 11:59 pm CST and is an individual assignment. All problems must be handwritten. Scan your work and submit a PDF file. For all probability computations, use the table at the end of this assignment.

Note: All interest rates and dividend yields are expressed per year with continuous compounding.

Problems

Impact of Dividends

Problem 1. Tupungato Investments specializes in asset management of commercial real estate properties. The stock pays a dividend yield of 4% per year, and its price is $200. Over the next six months, investors believe that there is an equal probability that it could increase to $221 or decrease to $164. What is the price of a six-month at-the-money European put option written on the stock? Use an interest rate of 8% per year in your computations.

Problem 2. The stock of Boulder Corp. trades at $50 per share and pays a dividend yield of 8% per year with continuous compounding. The stock could trade at $70 or $40 in six months, depending on how the economy evolves. The risk-free rate is currently 5% per year. A banker just sold to a client an instrument that pays in six months $100 if the stock price goes up and $200 otherwise. How many shares of the stock (per instrument sold) does she need to buy or sell to hedge her exposure?

Problem 3. A stock that pays a dividend yield of 5% per year costs $100. Financial analysts at all major investment banks agree there is a 60% probability that the stock will trade for $150 next year if the company succeeds in developing a novel AI algorithm. Otherwise, the stock price could fall to $50. The risk-free rate is 5% per year and is expected to remain constant. What should be the price of a security that pays $100 next year if the stock goes up and $0 otherwise?

American Options

Problem 4. The current price of a non-dividend-paying stock is \$150. Every three months, it is expected to go up or down by 12% or 16%, respectively. The risk-free rate is 10% per year. Compute the price of an American put option with a strike price of \$155 and a maturity of six months written on the stock.

Problem 5. The current price of a stock is \$150. Every six months, it is expected to go up or down by 19% or 15%, respectively. The stock pays a dividend yield of 6% per year and the risk-free rate is 8% per year with continuous compounding. Compute the price of a one-year American call option with strike price \$145 written on the stock.

Problem 6. *A friend of yours, who is attending another prestigious university, says: "I learned in class that an American call option written over an asset that pays a dividend yield $\delta > 0$ may only be exercised early if the interest rate is positive and the underlying asset price is high enough, but never if the interest rate is negative."* Determine whether the statement is true or false and explain to your friend why they should have chosen WashU instead.

The Log-Normal Distribution

Problem 7. Suppose that X is a normally distributed variable with mean $\mu = 4.30$ and standard deviation $\sigma = 1.20$. If $Y = e^X$ compute the following quantities.

a. The probability that Y is greater than 50?
b. The probability that Y is less than 80?
c. The probability that Y is between 60 and 70?

Statistics of Financial Markets

Problem 8. You are analyzing TMB stock. You believe that it is accurate to model the price evolution of the stock as a geometric Brownian motion. Using historical data, you estimate that the drift (μ) is 10.8% per year and the volatility of stock returns (σ) is 30% per year. The stock price just closed at \$229. Compute the expected stock price in 12 months from now.

Problem 9. You would like to invest in Abacus Corp. but are concerned that the stock price might drop. You have been studying the dynamics of the stock price and concluded that the stock follows a geometric Brownian motion with drift (μ) equal to 15% per year and volatility of price returns (σ) of 62% per year. The current stock price is \$102. Compute the probability that the stock price is less than \$92 in 10 months from now.

Problem 10. Krypto Corp. has been gaining much attention in the media for its upside potential. Financial experts agree that the stock price follows a geometric Brownian motion with drift (μ) equal to 18% per year and volatility of price returns (σ) of 49% per year. The current stock price is \$193. Compute the probability that the stock price is greater than \$224 in 12 months from now.

Problem 11. In class, we wrote

$$\frac{dS}{S} = \mu dt + \sigma dW$$

to describe the evolution of the stock price S of a non-dividend-paying stock over time. In that expression, what does the term $\frac{dS}{S}$ represent?

Problem 12. Let S be the price of NVDA stock that follows a geometric Brownian motion such that $dS = \mu S dt + \sigma S dW$. Your sales team would like to launch a new product called HEX that tracks the price of NVDA to the power 6. This would align well with the future line of graphics that NVIDIA is planning to launch. In other words, the value of this instrument is given by $Y = S^6$. Traders at your bank would like to know the dynamics of Y, since they would also like to write options on HEX, and make money selling them to important clients. Write down the dynamics of Y, that is, determine what $\frac{dY}{Y}$ looks like.

Table of Standard Normal Cumulative Probabilities

The following table reports values for $P(Z \leq z)$, where $Z \sim \mathcal{N}(0,1)$. If z is between two values, choose the one that is closer.

z	$P(Z \leq z)$	z	$P(Z \leq z)$	z	$P(Z \leq z)$
-2.32	0.0102	-0.72	0.2358	0.88	0.8106
-2.22	0.0132	-0.62	0.2676	0.98	0.8365
-2.12	0.0170	-0.52	0.3015	1.08	0.8599
-2.02	0.0217	-0.42	0.3372	1.18	0.8810
-1.92	0.0274	-0.32	0.3745	1.28	0.8997
-1.82	0.0344	-0.22	0.4129	1.38	0.9162
-1.72	0.0427	-0.12	0.4522	1.48	0.9306
-1.62	0.0526	-0.02	0.4920	1.58	0.9429
-1.52	0.0643	0.08	0.5319	1.68	0.9535
-1.42	0.0778	0.18	0.5714	1.78	0.9625
-1.32	0.0934	0.28	0.6103	1.88	0.9699
-1.22	0.1112	0.38	0.6480	1.98	0.9761
-1.12	0.1314	0.48	0.6844	2.08	0.9812
-1.02	0.1539	0.58	0.7190	2.18	0.9854
-0.92	0.1788	0.68	0.7517	2.28	0.9887

z	$P(Z \leq z)$	z	$P(Z \leq z)$	z	$P(Z \leq z)$
-0.82	0.2061	0.78	0.7823	2.32	0.9898