Prof. Lorenzo Naranjo Spring 2025

Options Spreads

Bull Spread

A bull spread is a two-leg option strategy that consists in a long position in a call with strike K_1 and a short position in a call with strike K_2, where $K_1 < K_2$.

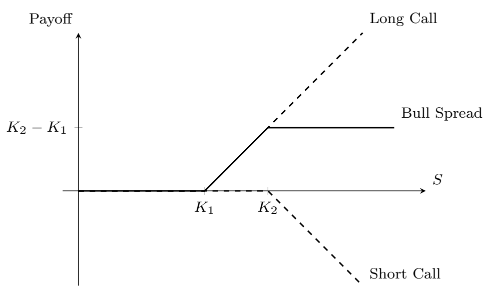

Figure 1: Payoff function of a bull spread strategy.

If $K_2 - K_1$ is small, the bull spread is like an all-or-nothing bet on the stock going above K_2.

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Warning: A numeric `legend.position` argument in `theme()` was deprecated in ggplot2
3.5.0.
i Please use the `legend.position.inside` argument of `theme()` instead.
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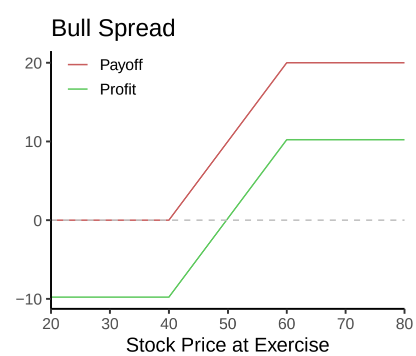
Example 1. A non-dividend paying stock currently trades at \$50. Call options with strikes \$40 and \$60 trade for \$13.23 and \$3.45, respectively. A bull spread that goes long the call with strike \$40 and shorts the call with strike \$60 costs 13.23 - 3.45 = \$9.78. Below are some possible bull spread payoffs and profits for different stock prices at maturity:



Stock Price	30	40	50	60	70
Payoff	0	0	10	20	20
Profit	-9.78	-9.78	0.22	10.22	10.22

The bull spread caps the maximum gains and losses at 20 - 9.78 = \$10.22 and 0 - 9.78 = -\$9.78, respectively.

We can derive the payoff table of the bull spread by combining the payoffs of the long call with strike price K_1 and the short call with strike price $K_2 > K_1$.

	$S \leq K_1$	$K_1 < S \leq K_2$	$K_2 < S$
Long Call	0	$S - K_1$	$S - K_1$
Short Call	0	0	$-(S - K_2)$
Bull Spread	0	$S - K_1$	$K_2 - K_1$

The previous analysis shows that the payoff of the bull spread is either zero or positive. Thus, no-arbitrage implies that the cost of a bull spread cannot be negative, that is,

$$C_1 - C_2 \geq 0.$$

If this were not the case, you could build a bull spread with a negative cost! This proves that a call with a lower strike cannot cost less than an otherwise equivalent call with a higher strike price, so that

$$C_1 \geq C_2.$$

In other words, if we keep everything else constant, the call premium is a decreasing function of the strike price. [1]

[1] In mathematics, we use partial derivatives to analyze changes in a variable while keeping everything else constant. Therefore, assuming that the call premium $C(K)$ is a differentiable function of the strike K We

Bear Spread

A bear spread is a two-leg option strategy that consists in a long position in a put with strike K_2 and a short position in a put with strike K_1, where $K_1 < K_2$.

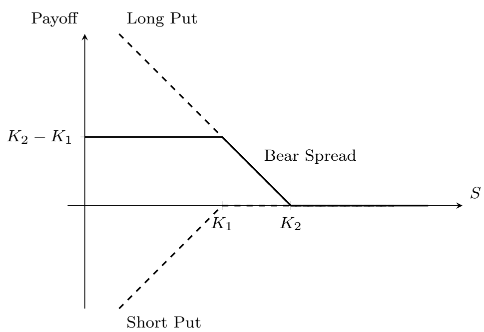
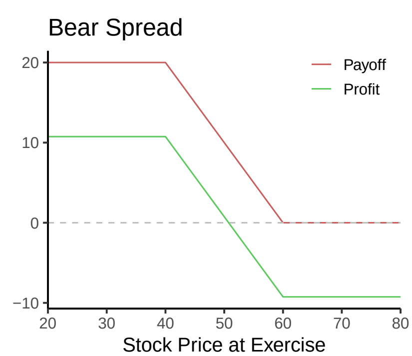


Figure 2: Payoff function of a bear spread strategy.

If $K_2 - K_1$ is small, the bear spread is like an all-or-nothing bet on the stock going below K_1.



Example 2. A non-dividend paying stock currently trades at \$50. Put options with strikes $K_1 = \$40$ and $K_2 = \$60$ trade for \$1.28 and \$10.53, respectively. A bear spread that goes long the put with strike K_2 and shorts the put with strike K_1 costs 10.53 - 1.28 = \$9.25. Below are some possible bear spread payoffs and profits for different stock prices at maturity:

Stock Price	30	40	50	60	70
Payoff	20	20	10	0	0
Profit	10.75	10.75	0.75	-9.25	-9.25

could write the previous statement as

$$\frac{\partial C(K)}{\partial K} < 0,$$

which is equivalent to say that the function is decreasing in the strike price.

The bear spread caps the maximum gains and losses at 20 - 9.25 = $10.75 and 0 - 9.25 = -$9.25, respectively.

As we did with the bull spread, we can derive the payoff table of the bear spread by writing down the payoffs of the long and short puts.

	$S \leq K_1$	$K_1 < S \leq K_2$	$S > K_2$
Long Put	$K_2 - S$	$K_2 - S$	0
Short Put	$-(K_1 - S)$	0	0
Bear Spread	$K_2 - K_1$	$K_2 - S$	0

The payoff table of the bear spread shows that the strategy can either pay nothing, or a positive amount. Thus, no-arbitrage implies that the cost of a bear spread cannot be negative, that is,

$$P_2 - P_1 \geq 0.$$

If not, you could build a bear spread with a negative cost! This implies that a put with a higher strike must cost more than an otherwise equivalent put with a lower strike price:

$$P_2 \geq P_1.$$

In words, keeping everything else constant, the put premium is an inceasing function of the strike price.

Butterfly

A butterfly is a three-leg option strategy that consists in a long call with strike K_1, short two calls with strike K_2 and a long call with strike K_3 where $K_1 < K_2 < K_3$ and $K_2 = (K_1 + K_3)/2$. The figure below shows the payoff of the different long and short calls, as well as the resulting buttefly.

The figure shows that the butterfly pays off if the stock price stays around K_2. In that sense the buttefly looks similar to a

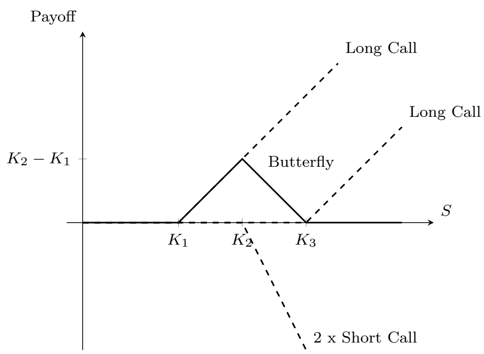
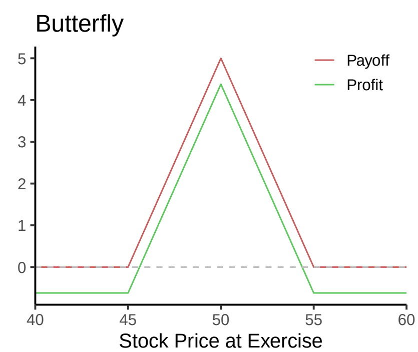


Figure 3: Payoff function of a butterfly strategy.

short straddle. The butterfly, though, unlike a short straddle is a debit position, meaning that you must pay something to establish it. This allows traders to take the same exposure as with a short straddle but limiting the overall exposure to whatever the butterfly costs.



Example 3. A non-dividend paying stock currently trades at \$50. Call options with strikes \$45, \$50 and \$55 trade for \$9.85, \$7.12 and \$5.01, respectively. A butterfly with strikes \$45, \$50 and \$55 then costs 9.85 - 2(7.12) + 5.01 = \$0.63. Below are some possible straddle payoffs and profits for different stock prices at maturity:

Stock Price	35	40	45	50	55	60	65
Payoff	0	0	0	5	0	0	0
Profit	-0.62	-0.62	-0.62	4.38	-0.62	-0.62	-0.62

The butterfly makes a profit if the stock stays very close to \$50. The return of getting the bet right is big. If the stock price ends up at \$50 at maturity then you would make 4.38/0.62 = 706% on your investment!

The payoff of a butterfly can then be described as follows. The first long call pays off whenever $S > K_1$, whereas the two short calls lose if $S > K_2$. Finally, the third long call pays off whenever $S > K_3$.

	$S \leq K_1$	$K_1 < S \leq K_2$	$K_2 < S \leq K_3$	$S > K_3$
Long Call	0	$S - K_1$	$S - K_1$	$S - K_1$
2 x Short Call	0	0	$2(K_2 - S)$	$2(K_2 - S)$
Long Call	0	0	0	$S - K_3$
Butterfly	0	$S - K_1$	$K_3 - S$	0

Note that the butterfly can also be obtained by buying puts with strikes K_1 and K_3, and shorting two puts with strikes $K_2 = (K_1 + K_3)/2$.

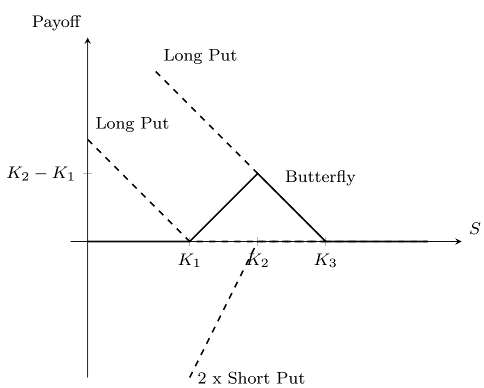


Figure 4: The butterfly can also be created using put options.

The figure shows that the payoff obtained using puts is the same as the one obtained when using call options. Also, the payoff of the butterfly is non-negative, which implies that the cost of the butterfly must be positive. Combining these two observations, no-arbitrage then implies that:

$$P_1 - 2P_2 + P_3 = C_1 - 2C_2 + C_3 \geq 0,$$

which in turn implies that $P_2 \leq \dfrac{P_1 + P_3}{2}$ and $C_2 \leq \dfrac{C_1 + C_3}{2}$. In words, this means that both the call and the put are convex functions in the strike price. This is because the average of the two extreme values is higher than the value of the put or call in the middle.

Condor

Similar to the butterfly, a condor is a four-leg option strategy that consists in a long call with strike K_1, a short call with strike K_2, a short call with strike K_3 and a long call with strike K_4 where $K_1 < K_2 < K_3 < K_4$ with $K_2 - K_1 = K_4 - K_3$.

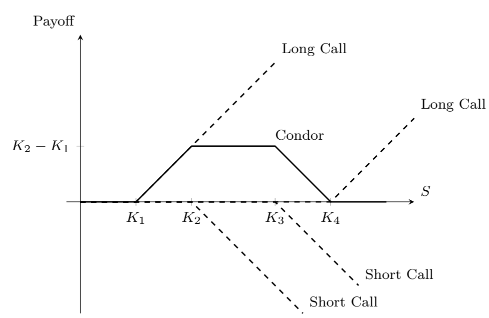


Figure 5: Payoff function of a condor strategy.

Example 4. A non-dividend paying stock currently trades at \$50. Call options with strikes \$40, \$45, \$55 and \$60 trade for \$13.23, \$9.85, \$5.01 and \$3.45, respectively. A condor build using those strikes then costs 13.23 - 9.85 - 5.01 + 3.45 = \$1.82. Below are some possible straddle payoffs and profits for different stock prices at maturity:

S	35	40	45	50	55	60	65
Payoff	0	0	5	5	5	0	0
Profit	-1.82	-1.82	3.18	3.18	3.18	-1.82	-1.82

The condor makes a profit if the stock stays between \$41.82 and \$58.18.

Practice Problems

Solutions to all problems can be found at lorenzonaranjo.com/fin451.

Problem 1. Suppose that put options on a stock with strike prices \$30 and \$35 cost \$4 and \$7, respectively. How can the options be used to create a bull spread and a bear spread? Construct a table that shows the profit and payoff for both spreads as a function of S when $S \leq 30$, $30 < S \leq 35$ and $S > 35$.

Problem 2. Three put options on a stock have the same expiration date and strike prices of \$55, \$60, and \$65. The market prices are \$3, \$5, and \$8, respectively. Explain how a butterfly spread can be created. Construct a table showing the profit from the strategy as a function of S. For what range of stock prices would the butterfly spread lead to a loss?

Problem 3. Suppose that put options on a stock with strike prices \$30 and \$35 cost \$4 and \$7, respectively. How can the options be used to create a bear spread?

Problem 4. Suppose you purchase one call option written on stock WFM, expiring in May, with strike price \$100, for \$5. At the same time, you write one call on WFM, expiring in May, with strike \$105, for \$2. If at expiration the price of a share of WFM stock is \$103, compute the profit per share.