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Options Markets

Definitions

An *option* is a type of derivative contract which gives the right, but not the obligation, to buy or sell a financial asset at a future date for a pre-determined price. The asset on which the option is written is called the *underlying* asset and in these notes its price is denoted by S.

Options can be written on a variety of underlying assets such as stocks, exchange-traded funds (ETFs) and notes (ETNs), stock market indices, futures, currencies, commodities, interest rates, bonds and swaps. In addition, options can also be embedded in other financial assets such as bonds or swaps. The *payoff* of an option then depends on the value of the underlying asset.

Since the option gives its holder the ability to *choose* whether or not to exercise the contract, having the option to buy or sell the asset requires the option holder to pay for it. The option's price per unit of underlying asset is called the option *premium*. The known price at which the option can be exercised is called the *strike* or *exercise* price, and is denoted by K.

There are two main type of options. A *call* option gives the holder the right to buy an asset by a certain date for the strike price whereas a *put* option gives the holder the right to sell an asset by a certain date for the exercise price.

It is important to note that only the buyer of the option can choose whether or not to exercise the contract. As such, the payoff to the buyer cannot be negative. The party that buys the option holds the *long* position whereas the seller or *writer* of the option holds the *short* position. The writer of the option might consequently face losses should the buyer exercise the contract.

The date specified in the contract is called the *expiration* or *maturity* date. The time remaining until the expiration date is called *time-to-maturity* and is denoted by T. Some options allow the buyer to exercise them prior to maturity, whereas other types of options can only be exercised at maturity.

An *American* type option can be exercised anytime up to the expiration date. Most options traded on exchanges are American, such as options on stocks and futures. It is in general hard, though, to determine when it is optimal to exercise early.

A *European* type option can be exercised only at maturity. European options are found, for example, in the OTC currency market, and most recently in futures exchanges as well. It is easier to analyze and price European options, although the methods we will learn in this class will apply for American type options as well.

Options Payoffs

We will now study the cash flows generated by call and put options. The option's *payoff* is defined as the cash flow generated by the derivative. The payoff is non-negative to the option's buyer, and negative or zero to the option's seller.

The option's *profit* is defined as the payoff minus its cost. Since the buyer of the option pays the option premium to its seller, we will take the convention that the cost is positive to the buyer and negative to the seller. Therefore, the option's writer makes a profit when the contract payoff is zero.

Let us analyze first the payoff of a long call option. Remember that a call option gives its holder the right but not the obligation to purchase the stock for a price K. If at maturity the price of the stock is $S > K$ then it makes sense to purchase the asset for K and sell it immediately for S, generating a cash flow of $S - K > 0$. On the contrary, if the stock price at maturity is $S \leq K$, then it does not make sense to exercise the call and the payoff is 0.

Therefore, the payoff of the call can be described as:

$$\text{Call Payoff} = \begin{cases} S - K & \text{if } S > K \\ 0 & \text{if } S \leq K \end{cases}$$

Therefore, the call payoff is the greatest between $S - K$ and 0, which also allows us to write:

$$\text{Call Payoff} = \max(S - K, 0).$$

This is a common way to express the payoff of a call option, which is useful when generating a plot using a spreadsheet software or any other programming language.

Example 1. Consider a stock call option with maturity 1 year, strike price $100 and currently trading at $14.

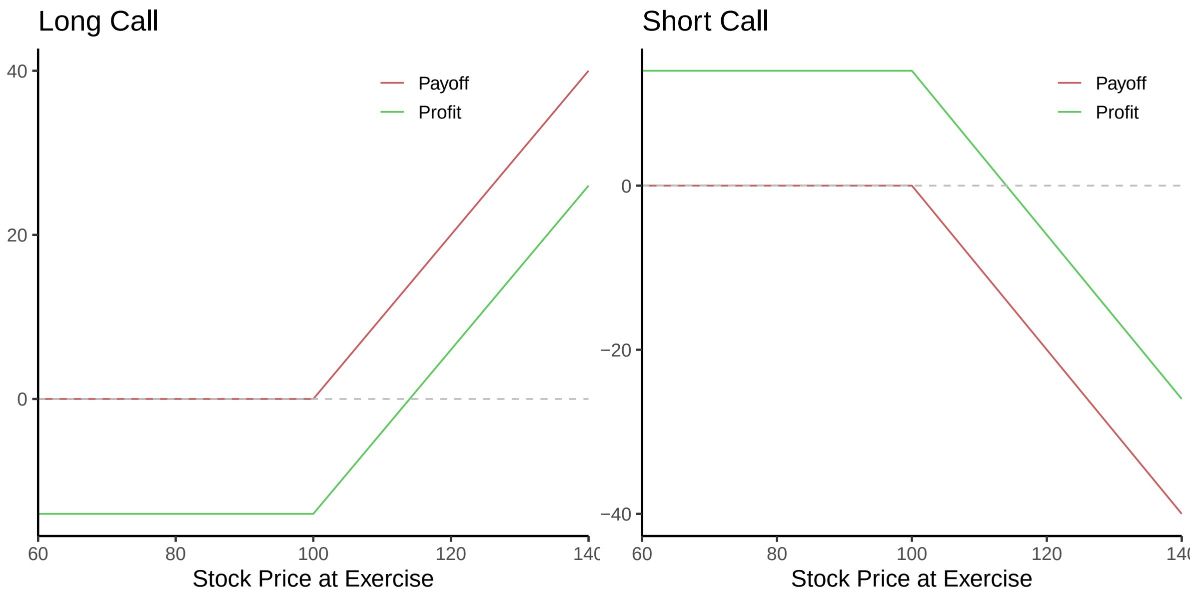
If the stock price at expiration is $120, then it makes sense for the buyer of the option to exercise it and purchase the stock for $100. In that case the payoff for the buyer is $120-100 = \$20$, generating a profit of $20-14 = \$6$, or equivalent a return on investment of $6/14 = 42.9\%$.

If on the other hand the stock price at expiration is $80, then it does not make sense to exercise the option and pay $100 for an asset that trades for $80. The payoff is therefore $0 and the profit is $0 - 14 = -\$14$. The return on investment in this case is $-14/14 = -100\%$. In other words, if the call at maturity is not exercised that generates a total loss of the initial premium paid to purchase the option.

We can compute the payoff, profit and return (per share) for different values of the stock price at exercise.

Stock Price	60	80	100	120	140
Payoff	0	0	0	20	40
Profit	-14	-14	-14	6	26
Return (%)	-100	-100	-100	42.9	185.7

The figure below plots the payoff and profit diagram of the long position in the call option.



The payoff and profit diagram of the short position is the mirror image of the long position with respect to the x-axis. □

We now turn our attention to put options, which give the holder the right but not the obligation to sell a stock for K. If at maturity the stock price is $S < K$, then it makes sense to exercise the put. Indeed, we can then purchase the shares for S and sell then for K, generating a cash flow of $K - S > 0$. If on the other hand at maturity we have that $S \geq K$, then it does not make sense to exercise the put and the payoff is zero.

The payoff function of the put is therefore:

$$\text{Put Payoff} = \begin{cases} K - S & \text{if } S < K \\ 0 & \text{if } S \geq K \end{cases}$$

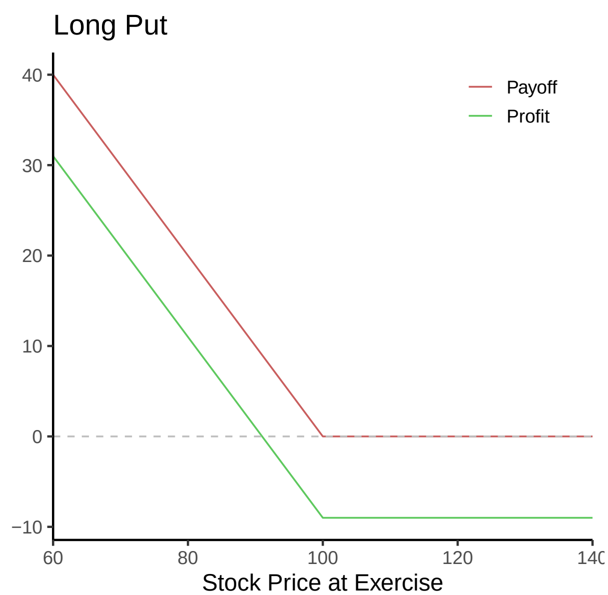
or equivalently,

$$\text{Put Payoff} = \max(K - S, 0).$$

Example 2. Consider a stock put option with maturity 1 year, strike price \$100 and currently trading at \$9. If the stock price at expiration is \$80, then it makes sense for the buyer of the option to exercise it and sell the stock for \$100. The payoff for the buyer is $100 - 80 = \$20$ generating a profit of $20 - 9 = \$11$. The return on investment is $11/9 = 122.22\%$.

If on the other hand the stock price at expiration is \$120, it does not make sense to exercise the option and get \$100 for an asset that trades for \$120. The payoff is therefore \$0 and the profit is $0 - 9 = -\$9$, or equivalently $-9/9 = -100\%$.

As before, we can plot the payoff and profit of a long put option.



We can see that the put has a maximum payoff of \$40. □

The table below summarizes the payoff functions of long and short positions for call and put options.

	Long	Short
Call	$\max(S - K, 0)$	$-\max(S - K, 0)$
Put	$\max(K - S, 0)$	$-\max(K - S, 0)$

We can alternatively plot the payoff diagrams of long and short positions for call and put options.

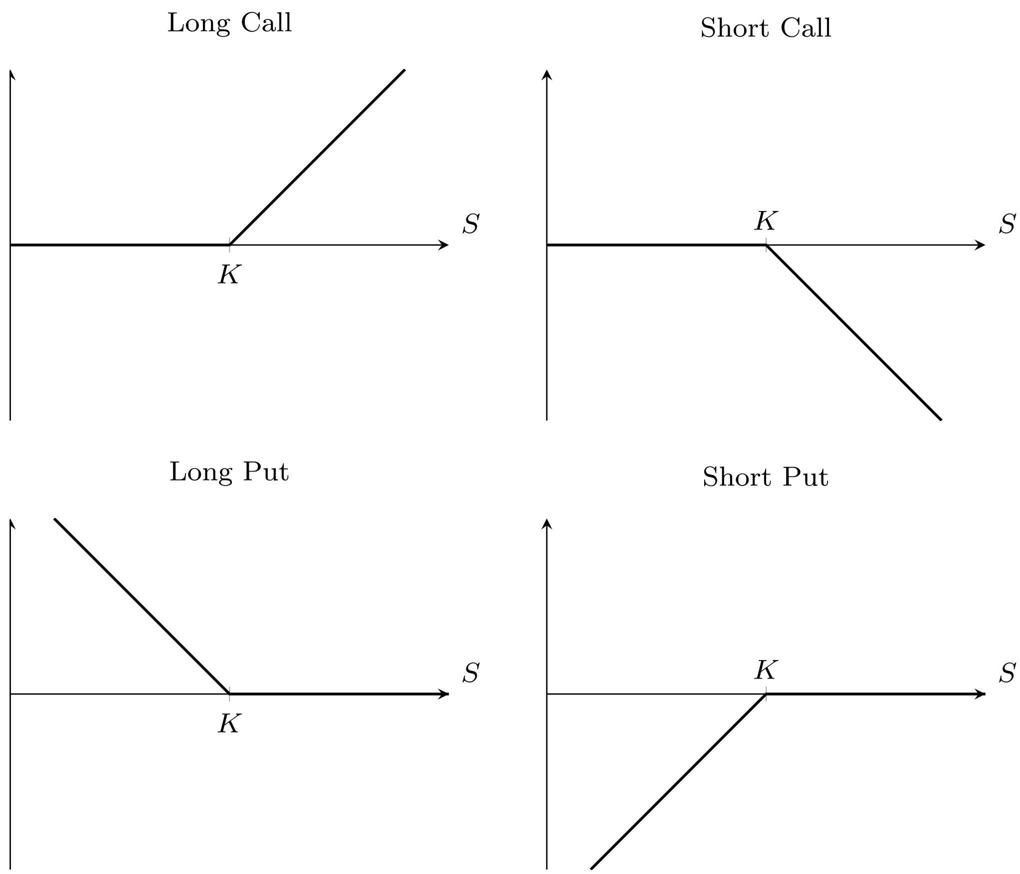


Figure 1: Payoff diagrams for long and short positions in call and put options.

Further Definitions

Option Moneyness

The moneyness of an option indicates whether the payoff of a long position at a certain point in time is positive or not. An option is said to be *in-the-money* (ITM) if it is profitable to exercise it immediately, *at-the-money* (ATM) if the strike is equal to the current spot price, and *out-of-the-money* (OTM) if it is not profitable to exercise it immediately.

The following table describes the moneyness of call and put options based on the level of the spot price.[1]

	Call	Put
Out-of-the-money	$K > S$	$K < S$
At-the-money	$K = S$	$K = S$
In-the-money	$K < S$	$K > S$

Contract Size

Call and put option contracts are in general written over several units of the underlying asset, such as 100 shares, but their prices are quoted per unit of the underlying asset. For example, consider a put option contract on 100 shares of AAPL stock with strike $110. If the current stock price is $122 and the price of an option to sell one share in 3 months is $0.85, the payoff and profit of a contract (100 shares) for different values of the spot price at maturity is:

Stock Price	80	90	100	110	120	130
Payoff	3,000	2,000	1,000	0	0	0
Profit	2,915	1,915	915	-85	-85	-85
Return (%)	3,429	2,253	1,076	-100	-100	-100

Traded Volume vs. Open Interest

For both call and put options, for every long position there is a corresponding short position, i.e., the contracts are in zero net-supply. The total number of long positions, which

[1] It is customary in currency markets to define the moneyness of options with respect to forward prices. Therefore, an *at-the-money-forward* (ATMF) option is one in which the strike is equal to the forward price with the same maturity as the option.

is the same as the total number of short positions, is called open-interest. The traded volume on the other hand is the number of contracts that are bought or sold.

Note that a trader can buy a contract, then sell it the same day. The volume for that day would be 2 but the open-interest would not change.

Traders look carefully at the ratio of long puts vs. calls in the S&P 500 which is commonly known as the put/call ratio.

Intrinsic and Time Value

The intrinsic value of an option is the payoff that the buyer would get if the option was exercised at that time:

$$\text{Intrinsic Value} = \begin{cases} \max(S-K,0) & \text{for a call} \\ \max(K-S,0) & \text{for a put} \end{cases}$$

The *time value* of the option is defined as the difference between the option premium and its intrinsic value at that point in time, i.e.

$$\text{Time Value} = \text{Option Premium} - \text{Intrinsic Value}.$$

We will see later that even though the time value of an American options is always positive or zero, the time value of a European option might be negative.

Practice Problems

Problem 1. Explain why brokers require margins when clients write options but not when they buy options.

Problem 2. Explain why an American option is always worth at least as much as a European option on the same asset with the same strike price and exercise date.

Problem 3. Why an American put option is always worth at least its intrinsic value?

Problem 4. The table below list prices at close on September 1st, 2021 for various options expiring in November written on Apple Inc. (AAPL). The underlying stock price on that date closed at $152.51.

Strike	Call Last	Volume	Open Interest	Put Last	Volume	Open Interest
140	15.35	1,678	9,117	2.9	998	16,928
145	11.62	1,478	9,310	4.24	929	15,310
150	8.58	3,757	40,111	6.15	3,218	9,004
155	6.15	5,700	23,044	8.64	1,822	2,786
160	4.2	5,168	30,776	11.45	191	2,963

Use the data in the table to calculate the payoff and the profit for investments in each of the following November expiration options, assuming that the stock price on the expiration date is $162.

Type	Strike	Payoff	Profit
Call	140		
Put	140		
Call	145		
Put	145		
Call	150		
Put	150		
Call	155		
Put	155		
Call	160		
Put	160		

Problem 5. An investor buys 500 shares of a stock and sells five call option contracts on the stock. Each contract is for 100 shares. The strike price is $30. The price of the option is $3. Compute the investor's minimum cash investment if the stock price is:

a. $28
b. $32

Problem 6. On September 7, 2021, XYZ stock closed at $102. A December call option on XYZ with strike $100 closed at $8 per share. If the stock price at maturity is $95, compute the profit per share of investing in such a call option.

Problem 7. On September 7, 2021, ABC stock closed at $55. A January put option on ABC with strike $51 closed at $8 per share. If the stock price at maturity is $36, compute the profit per share of investing in such a put option.

Problem 8. On September 7, 2021, XYZ stock closed at $93. A December call option on XYZ with strike $51 closed at $10 per share. If the stock price at maturity is $53, compute the payoff per share that an investor will receive for investing in such a call option.

Problem 9. An investor buys 418 shares of a stock and sells 9 call option contracts on the stock. The stock currently trades for $102. Each contract is for 100 shares with strike price $101, and the price of each option is $8 per share. Compute the cost of the strategy.

Problem 10. Suppose the price of a share of Google stock is $500. An April call option on Google stock has a premium of $5 and an exercise price of $500. Ignoring commissions, the holder of the call option will earn a profit if the price of the share:

a. increases to $504.
b. decreases to $490.
c. increases to $506.
d. decreases to $496.
e. None of the options