Options on Futures

Definitions

The underlying asset is a futures contract, usually on a commodity, a precious metal, a currency, an interest rate or a bond. These options usually expire on or a few days before the earliest delivery date of the underlying futures contract. If a call futures option is exercised, the buyer gets a long position in the futures plus a cash amount equal to the excess of the futures price at the time of the most recent settlement over the strike price. If a put futures option is exercised, the buyer acquires a short position in the futures plus a cash amount equal to the excess of the strike price over the futures price at the time of the most recent settlement.

Example 1. September call option contract on copper futures has a strike of 425 cents per pound. A CME contract is written on 25,000 pounds of copper. The call is exercised when the futures price is 426.95 cents and the most recent settlement is 426.45 cents. The trader receives a long September futures contract on copper and $25,000 \times (4.2645 - 4.2500) = \$362.50$. □

Example 2. A September put option contract on soybean futures has a strike price of 1,380 cents per bushel. A CME contract is written over 5,000 bushels. The put is exercised when the futures price is 1372 cents per bushel and the most recent settlement price is 1365 cents per bushel. The trader receives a short September futures contract on soybean and $5,000 \times (13.80 - 13.65) = \$750$ in cash. □

Futures contracts are easier to trade and more liquid than the underlying asset, as would be the case for commodities, for example. The exercise of the option does not lead to delivery of underlying asset, but of the futures contract on the asset which can be bought or sold easily. Futures options and futures usually trade on same exchange, i.e., Chicago Mercantile Exchange (CME). By trading on exchanges, futures options are more liquid and may entail lower transactions costs.

The Risk-Neutral Process for the Futures Price

We saw previously that the forward or futures price on an asset paying a dividend yield is given by[1]

$$F = Se^{(r-q)(T-t)}.$$

We can apply Ito's lemma to F to determine its dynamics under the *risk-neutral measure*:

$$\begin{aligned} dF &= \frac{\partial F}{\partial S}dS + \frac{1}{2}\frac{\partial^2 F}{\partial S^2}(dS)^2 + \frac{\partial F}{\partial t}dt \\ &= (r-q)Fdt + \sigma F dW - (r-q)Fdt \\ &= \sigma F dW^*. \end{aligned}$$

This shows that the futures price is a martingale under the risk-neutral measure.

Black's Model for European Futures Options

It is sometimes very useful in mathematics to introduce the right *zero* in an equation. We note that equation (1) remains true if we re-write it as:

$$dF = (r-r)Fdt + \sigma F dW$$

This means that we could interpret the futures price *as if* it was a tradable asset that provides a dividend yield equal to the risk-free rate. If this is the case, we could then apply the standard Black-Scholes formula for an asset that pays a dividend yield q.

Black's Model for European Futures Options

Consider a futures contract expiring at T written on an asset S that pays a continuous dividend yield δ and that follows a GBM under the risk-neutral measure:

$$dS = (r-\delta)Sdt + \sigma S dW.$$

The futures price at time t is given by:

$$F = Se^{(r-\delta)(T-t)},$$

and also follows a GBM under the risk-neutral measure,

$$dF = \sigma F dW.$$

[1] Remember that the value of a forward contract with delivery price K and expiring at T is:

$$V = Se^{-q(T-t)} - Ke^{-r(T-t)}$$

We can see that the value of the forward contract is zero is we set $K = F$.

The price of European call and put futures options with strike price K and time-to-maturity T are given by:

$$C = Fe^{-rT}\,\Phi(d_1) - Ke^{-rT}\,\Phi(d_2)$$
$$P = Ke^{-rT}\,\Phi(-d_2) - Fe^{-rT}\,\Phi(-d_1)$$

where $d_1 = \dfrac{\ln(F/K) + \frac{1}{2}\sigma^2 T}{\sigma\sqrt{T}}$ and $d_2 = d_1 - \sigma\sqrt{T}$.

Therefore, European futures options and spot options are equivalent when futures contract matures at the same time as the option.

Black's Model in Practice

Black's model is frequently used to value European options on the spot price of an asset in the over-the-counter market (OTC). This avoids the need to estimate income on the asset since we can use the futures price directly in the formula.

Example 3. Consider a 6-month European call option on spot gold. The 6-month futures price is $1,806, the 6-month risk-free rate is 1% per year continuously-compounded, the strike price is $1,820, and the volatility of the futures price is 20% per year. The option is priced using Black's model with $f_0 = 1806$, $K = 1820$, $r = 0.01$, and $\sigma = 0.20$:

$$\begin{aligned}
d_1 &= \frac{\ln(1806/1820) + \frac{1}{2}(0.20)^2(0.5)}{0.20\sqrt{0.5}} = 0.0161 \Rightarrow \Phi(d_1) = 0.5064 \\
d_2 &= 0.0161 - 0.20\sqrt{0.5} = -0.1253 \Rightarrow \Phi(d_2) = 0.4501 \\
C &= 1806e^{-0.01(0.5)}(0.5064) - 1820e^{-0.01(0.5)}(0.4501) = 94.88
\end{aligned}$$

The value of the call is $94.88. □

Futures Style Options

A futures-style option is a futures contract on a futures option. One of the advantages of these contracts is that the margining of the instrument is the one of a futures contract, which means that you do not have to pay the premium upfront but instead deposit a margin. Gains and losses are then marked to market daily and credited or debited from the margin account.

If we denote by ϕ_C the futures price for a call futures-style option and by C the price of the underlying futures option, we have that:

$$\phi_C = Ce^{rT} = F\,\Phi(d_1) - K\,\Phi(d_2)$$

Similarly, the futures price of a put futures-style option is:

$$\phi_P = Pe^{rT} = K\,\Phi(-d_2) - F\,\Phi(-d_1)$$

Practice Problems

Problem 1. A futures price is currently 25, it's volatility is 30% per year, and the risk-free rate is 10% per year. What is the value of a nine-month European call on the futures with a strike price of 26?

Problem 2. Calculate the price of a three-month European call option on the spot value of silver. The three-month futures price is $12, the strike is $13, the risk-free rate is 4% and the volatility of the price of silver is 25%.