Options, Futures and Derivative Securities
Time: 80 minutes

FIN 451
20 points

Mock Midterm 3

Questions

Problem 1 (3 pts). Suppose that the derivatives desk at Morgan Stanley has just sold 10,000 European puts to BlackRock. Each put is written on the MS-30 Tech Index, which tracks 30 high-growth technology companies. The index is currently at 4,500 points, and pays a dividend yield of 2% per year. The puts expire in one year, have a strike price of 4,100 and are cash settled at expiration. The risk-free rate is 4.5% per year with continuous compounding. The volatility desk estimates that the volatility of the index returns is 45% and expected to remain constant for the next year.

a. There's an ETF (ticker: MSTX) that tracks the index perfectly and currently trades for $130. How many shares of the ETF does the trader need to buy/sell initially in order to hedge the exposure created by the sale of the puts?
b. How much money does the trader need to borrow/lend today in order to make sure that the strategy is self-financing?
c. When hedging the puts, should the trader be more worried about gamma or vega risk?

Problem 2 (3 pts)**.** Suppose that the FX Trading desk at Goldman Sachs is analyzing a EUR/USD position for a sovereign wealth fund client. The spot price of the Euro (EUR) is USD 1.15 and the EUR/USD exchange rate has a volatility of 4% per annum. The ECB benchmark rate in Europe is 2.25% per year whereas the Fed Funds rate in the United States is 4.50% per year.

a. Calculate the value of a European option to sell EUR 100,000,000 and receive USD 110,000,000 in six months. This represents a notional-weighted strike of 1.10 USD per EUR.
b. Use put-call parity to calculate the price of a European option to buy EUR 100,000,000 for USD 110,000,000 in six months. The client is considering this call as an alternative hedging strategy for their upcoming European acquisition.
c. Explain why the Black-Scholes formula to buy €1 at time T for a predetermined exchange rate K is given by

$$C = Fe^{-rT}\,\Phi(d_1) - Ke^{-rT}\,\Phi(d_2),$$

where $d_1 = \dfrac{\ln(F/K) + \frac{1}{2}\sigma^2 T}{\sigma\sqrt{T}}$, and $d_2 = d_1 - \sigma\sqrt{T}$.

Note: The table below might come handy to compute $\Phi(-d_1)$ and $\Phi(-d_2)$.

z	P(Z ≤ z)	z	P(Z ≤ z)	z	P(Z ≤ z)	z	P(Z ≤ z)
-2.00	0.0228	-1.97	0.0244	-1.94	0.0262	-1.91	0.0281
-1.99	0.0233	-1.96	0.0250	-1.93	0.0268	-1.90	0.0287
-1.98	0.0239	-1.95	0.0256	-1.92	0.0274	-1.89	0.0294

Problem 3 (2 pts). Consider a European put option expiring in 6 months and with strike price equal to $103, written on a stock that currently trades for $100. Interestingly, the volatility of the stock is zero. The risk-free rate is 5% per year with continuous compounding and the stock pays a dividend yield of 2% per year.

a. Compute the price of the put option.
b. Does put-call parity hold if the volatility of the asset returns is equal to zero?

Problem 4 (2 pts). Consider a credit bear spread with strikes K_1 and $K_2 > K_1$ made using European call options written on a non-dividend paying asset and expiring in two months.

a. In separate diagrams, draw the price, delta and gamma of the bear spread as a function of the stock price.
b. Determine the sign of the theta if the stock price is equal to K_1 and K_2, respectively.

Problem 5 (2 pts). Consider a blue-chip tech stock in JP Morgan's equity derivatives portfolio that pays a dividend yield of 2% and has a volatility of returns of 45%. The stock price is $95 and the risk-free rate is 4.5%.

a. Compute the price of an asset-or-nothing put that pays 1 share of the stock if the stock price in one month is below $90. This exotic option was requested by a hedge fund client looking to implement a sophisticated collar strategy.
b. Compute the price of a cash-or-nothing put that pays $100 if the stock price in one month is below $90. The trading desk is considering offering this binary option to complement the client's existing positions.

Problem 6 (2 pts). Calculate the price of a three-month European put option on Bitcoin futures expiring in three months. The three-month futures price is $89,215, the strike is $89,000, the risk-free rate is 4.50% and the volatility of the price returns of BTC is 85%.

Problem 7 (2 pts). Determine whether the following statements are true or false and briefly explain why.

a. A chooser option is very similar to a straddle since at the moment in which you can choose whether you want a call or a put you get pretty much what a straddle pays off.
b. In order to be able to price a forward-start option in closed-form it is crucial that the option starts at-the-money.

Problem 8 (4 pts). Consider a non-dividend paying stock that trades for $50. Every 3-months, the stock price can increase or decrease by 10%. The risk-free rate is 5% per year with continuous compounding. Compute the price of the following path-dependent options expiring in 6 months.

a. A floating lookback call that pays $S_T - S_{min}$ at maturity.
b. A floating lookback put that pays $S_{max} - S_T$ at maturity.
c. An average price Asian put option that pays $\max(50 - \bar{S}, 0)$ at maturity.
d. An average strike Asian call option that pays $\max(S_T - \bar{S}, 0)$ at maturity.

Formula Sheet

In the following, S denotes the stock or spot price of an asset, r is the continuously-compounded risk-free rate expressed per year, δ denotes the dividend yield, T denotes the time-to-maturity of a forward, futures or an option, and K denotes the strike price of an option or the delivery price of a forward contract.

Binomial Pricing

At any node of a binomial tree in which the stock price can move up to $S_u = u \times S$ or down to $S_d = d \times S$, the risk-neutral probability of an up-move is given by

$$q = \frac{Se^{(r-q)\Delta t} - S_d}{S_u - S_d} = \frac{e^{(r-q)\Delta t} - d}{u - d},$$

where Δt denotes the length of each period. To make the tree consistent with the observed volatility of stock returns, we typically choose $u = e^{\sigma\sqrt{\Delta t}}$ and $d = 1/u$.

Impact of Dividends Dividends

For European call and put options with strike price K and time-to-expiration T written on a non-dividend paying asset, we have that

$$C - P = Se^{-\delta T} - Ke^{-rT},$$

where C and P denote the call and put prices. Put-call parity implies the following bounds for European call and put options:

$$\max(Se^{-\delta T} - Ke^{-rT}, 0) \leq C \leq S,$$
$$\max(Ke^{-rT} - Se^{-\delta T}, 0) \leq P \leq Ke^{-rT}.$$

Pricing Formulas

In the Black-Scholes model where

$$dS = (r - \delta)Sdt + \sigma S dW,$$

we have the following results for European call and put options. In the formulas,

$$d_1 = \frac{\ln(S/K) + (r - \delta + \frac{1}{2}\sigma^2)}{\sigma\sqrt{T}},$$

and $d_2 = d_1 - \sigma\sqrt{T}$.

Variable	Call	Put
V	$Se^{-\delta T}\,\Phi(d_1) - Ke^{-rT}\,\Phi(d_2)$	$Ke^{-rT}\,\Phi(-d_2) - Se^{-\delta T}\,\Phi(-d_1)$
Δ	$e^{-\delta T}\,\Phi(d_1)$	$-e^{-\delta T}\,\Phi(-d_1)$
Γ	$\frac{e^{-\delta T}\,\Phi'(d_1)}{S\sigma\sqrt{T}} = \frac{Ke^{-rT}\,\Phi'(d_2)}{S^2\sigma\sqrt{T}}$	
Θ	$rV - (r-\delta)S\Delta - \frac{1}{2}\sigma^2 S^2 \Gamma$	
$\mathcal{V}$	$Se^{-\delta T}\,\Phi'(d_1)\sqrt{T} = Ke^{-rT}\,\Phi'(d_2)\sqrt{T}$	
ρ	$KTe^{-rT}\,\Phi(d_2)$	$-KTe^{-rT}\,\Phi(-d_2)$

The Standard Normal Distribution

The following table reports values for $\phi(z) = P(Z \leq z)$, where $Z \sim \mathcal{N}(0, 1)$.

z	P(Z ≤ z)	z	P(Z ≤ z)	z	P(Z ≤ z)	z	P(Z ≤ z)
-2.37	0.0089	-1.17	0.1210	0.03	0.5120	1.23	0.8907
-2.32	0.0102	-1.12	0.1314	0.08	0.5319	1.28	0.8997
-2.27	0.0116	-1.07	0.1423	0.13	0.5517	1.33	0.9082
-2.22	0.0132	-1.02	0.1539	0.18	0.5714	1.38	0.9162
-2.17	0.0150	-0.97	0.1660	0.23	0.5910	1.43	0.9236
-2.12	0.0170	-0.92	0.1788	0.28	0.6103	1.48	0.9306
-2.07	0.0192	-0.87	0.1922	0.33	0.6293	1.53	0.9370
-2.02	0.0217	-0.82	0.2061	0.38	0.6480	1.58	0.9429
-1.97	0.0244	-0.77	0.2206	0.43	0.6664	1.63	0.9484
-1.92	0.0274	-0.72	0.2358	0.48	0.6844	1.68	0.9535
-1.87	0.0307	-0.67	0.2514	0.53	0.7019	1.73	0.9582
-1.82	0.0344	-0.62	0.2676	0.58	0.7190	1.78	0.9625
-1.77	0.0384	-0.57	0.2843	0.63	0.7357	1.83	0.9664
-1.72	0.0427	-0.52	0.3015	0.68	0.7517	1.88	0.9699
-1.67	0.0475	-0.47	0.3192	0.73	0.7673	1.93	0.9732
-1.62	0.0526	-0.42	0.3372	0.78	0.7823	1.98	0.9761
-1.57	0.0582	-0.37	0.3557	0.83	0.7967	2.03	0.9788
-1.52	0.0643	-0.32	0.3745	0.88	0.8106	2.08	0.9812
-1.47	0.0708	-0.27	0.3936	0.93	0.8238	2.13	0.9834
-1.42	0.0778	-0.22	0.4129	0.98	0.8365	2.18	0.9854
-1.37	0.0853	-0.17	0.4325	1.03	0.8485	2.23	0.9871
-1.32	0.0934	-0.12	0.4522	1.08	0.8599	2.28	0.9887
-1.27	0.1020	-0.07	0.4721	1.13	0.8708	2.33	0.9901
-1.22	0.1112	-0.02	0.4920	1.18	0.8810	2.37	0.9911