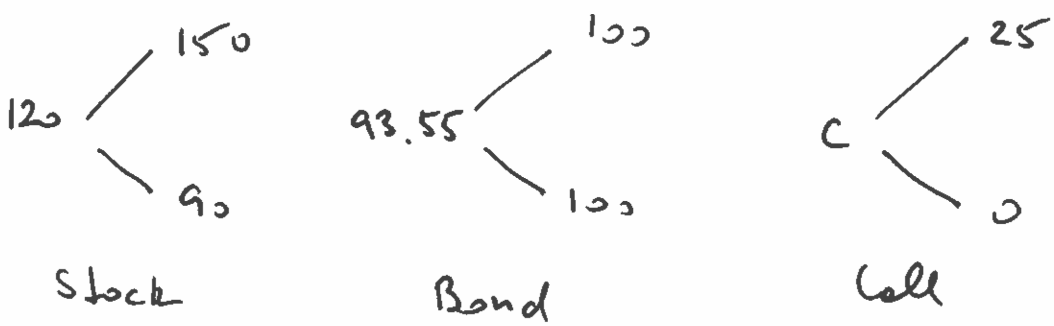
Options, Futures and Derivative Securities
Time: 80 minutes

FIN 451
20 points

Mock Midterm 2

Questions

Problem 1 (3 pts). A non-dividend paying stock price is currently \$120. It is known that at the end of 8 months it will be either \$150 or \$90. The risk-free interest rate is 10% per annum with continuous compounding. What is the value of an 8-month European call option with a strike price of \$125? Use the replicating portfolio argument and indicate the number of shares required to hedge the position. Use a risk-free bond with face value \$100 in your computations.



$$150 N_S + 100 N_B = 25$$
$$90 N_S + 100 N_B = 0 \leftarrow$$

$$N_S = \frac{25-0}{150-90} = 0.417$$

$$N_B = -\frac{90}{100} N_S = -0.9 \times 0.417 = -0.375$$

$$C = 120 \times 0.417 - \underbrace{93.55 \times 0.375}_{35.08} = 14.96$$

Problem 2 (3 pts). A stock price is currently $50. It is known that at the end of 4 months it will be either $60 or $40. Analysts at a prestigious bank have determined that there is a 70% that the stock will be $60 in four months. The risk-free interest rate is 8% per annum with continuous compounding. The stock pays a dividend yield of 4% per year.

a. What is the value of a 4-month European put option with a strike price of $52? Use the risk-neutral probability method to compute the value of the option.
b. Using put-call parity, determine the price of a European call with the same characteristics.

a.

Stock Put

$$q = \frac{50\, e^{(0.08-0.04)\times 4/12} - 40}{60 - 40} = 0.534$$

$$Put = (0\, q + 12(1-q))\, e^{-0.08\times 4/12}$$

$$= 5.44$$

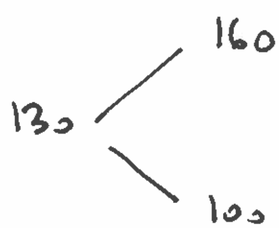
b. $$c - p = S\, e^{-\delta T} - K e^{-rT}$$

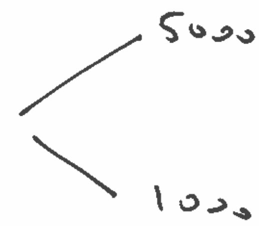
$$c = p + S e^{-\delta T} - K e^{-rT}$$

$$= 5.44 + 50\, e^{-0.04\times 4/12} - 52 e^{-0.08\times 4/12}$$

$$= 4.15$$

Problem 3 (2 pts). The stock of Boulder Corp. trades at \$130 per share and pays a dividend yield of 10% per year with continuous compounding. The stock could trade at \$160 or \$100 in six months, depending on how the economy evolves. The risk-free rate is currently 5% per year. A banker just sold to a client an instrument that pays in six months \$5,000 if the stock price goes up and \$1,000 otherwise. How many shares of the stock (per instrument sold) does she need to buy or sell to hedge her exposure?





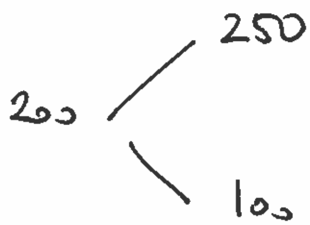
Stock

Instrument

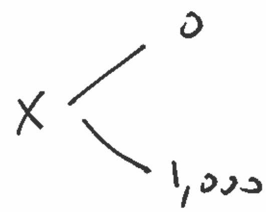
$$N_S = \left(\frac{5000 - 1000}{160 - 100}\right)\left(e^{-0.10 \times 6/12}\right) = 63.42$$

She need to buy 63 shares to hedge the exposure.

Problem 4 (2 pts). A stock that pays a dividend yield of 4% per year costs \$200. Financial analysts at all major investment banks agree there is a 60% probability that the stock will trade for \$250 next year if the company succeeds in developing a novel AI algorithm. Otherwise, the stock price could fall to \$100. The risk-free rate is 4% per year and is expected to remain constant. What should be the price of a security that pays \$0 next year if the stock goes up and \$1,000 otherwise?



Stock



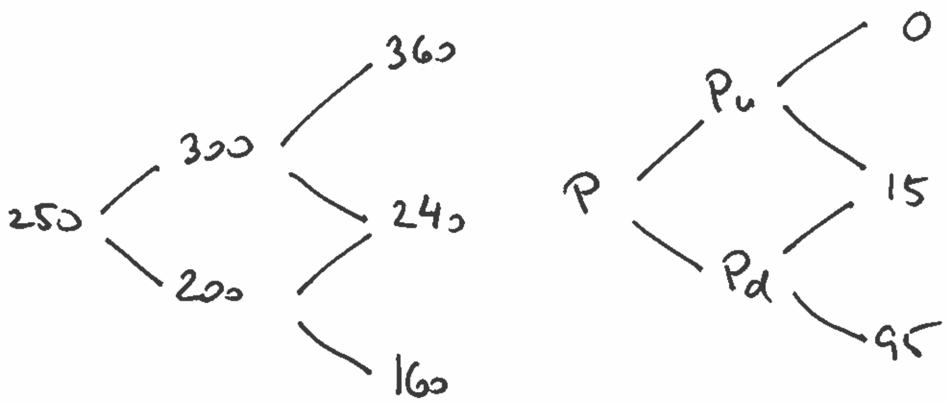
Instrument

$$q = \frac{200\, e^{(0.04 - 0.04)} - 100}{250 - 100} = \frac{100}{150} = \frac{2}{3}$$

$$X = (0\, q + 1000\, (1 - q))\, e^{-0.04}$$

$$= \frac{1}{3} * 1000\, e^{-0.04} = 320.26.$$

Problem 5 (4 pts). The current price of a non-dividend-paying stock is $250. Every three months, it is expected to increase or decrease by 20%. The risk-free rate is 10% per year. Compute the price of an American put option with a strike price of $255 and a maturity of six months written on the stock. In each node, explain why you would exercise the put or not.



Stock   Put

$$q = \frac{e^{0.10 \times 3/12} - 0.8}{1.2 - 0.8} = 0.563$$

If $S = 200$ Wait $= (15q + 95(1-q))e^{-0.10 \times 3/12} = 48.73$

Exercise $= 255 - 200 = \underline{55}$

$\boxed{P_d = 55}$

If $S = 300$ Wait $= (0q + 15(1-q))e^{-0.010 \times 3/12} = 6.39$

If $S = 250$ Wait $= (6.39q + 55(1-q))e^{-0.10 \times 3/12}$

$= 26.95$

Exercise $= 255 - 250 = 5$

$\Rightarrow \boxed{\text{Put} = 26.95}$

Problem 6 (2 pts). *A friend of yours, who is attending a prestigious university, says: "I learned in class that you should never exercise early an American call option written on a non-dividend paying asset. On the other hand, an American put option with similar characteristics should be exercised early if the stock price is low enough"* Determine whether the statement is true or false.

False. If $r<0$ you might to exercise the American call early but never the American put.

Problem 7 (4 pts). Let S be the price of GOOG stock that follows a geometric Brownian motion such that $dS = \mu S dt + \sigma S dW$. Your sales team would like to launch a new product called PWR that tracks the price of GOOG to the power 1.5. Traders at your bank are excited but would like to know the dynamics of $Y = S^{1.5}$, since they would also like to write options on PWR.

$$(dS)^2 = \sigma^2 S^2 dt$$

a. Write down the dynamics of Y, and determine its drift and volatility.

b. If GOOG stock is currently 160, compute the probability that PWR is higher than 2,220 in six months. In your computations, use $\mu = \underline{0.15}$ and $\sigma = \underline{0.50}$.

a. $Y = S^{1.5}$ $\quad Y_S = 1.5\, S^{0.5}$ $\quad Y_{SS} = 0.75\, S^{-0.5}$

$$dY = Y_S\, dS + \frac{1}{2} Y_{SS} (dS)^2$$

$$= 1.5\, S^{0.5} (\mu S\, dt + \sigma S\, dW) + \frac{1}{2} 0.75 S^{-0.5} \times \sigma^2 S^2 dt$$

$$= 1.5 \mu\, S^{1.5} dt + 1.5 \sigma S^{1.5} dW + 0.375 \sigma^2 S^{1.5} dt$$

$$\frac{dY}{Y} = \overbrace{(1.5\mu + 0.375\sigma^2)}^{0.319} dt + \overbrace{1.5\sigma}^{0.75} dW$$

b. If $S_0 = 160 \Rightarrow Y_0 = 160^{1.5} = 2023.86$

$$E \ln Y_T = \ln Y_0 + \left(0.319 - \frac{1}{2} 0.75^2\right) \times \frac{6}{12}$$

$$= \underline{7.632}$$

$$\text{St. Dev. } \ln Y_T = 0.75 \sqrt{6/12} = \underline{0.530}$$

$$\Pr(Y_T > 2220) = 1 - \Pr(Y_T \leq 2220)$$

$$= 1 - \Pr\left(Z \leq \frac{\ln(2220) - 7.632}{0.530}\right)$$

$$= 1 - \Pr(Z \leq 0.14) = 1 - 0.5517$$

$$= 0.4483$$

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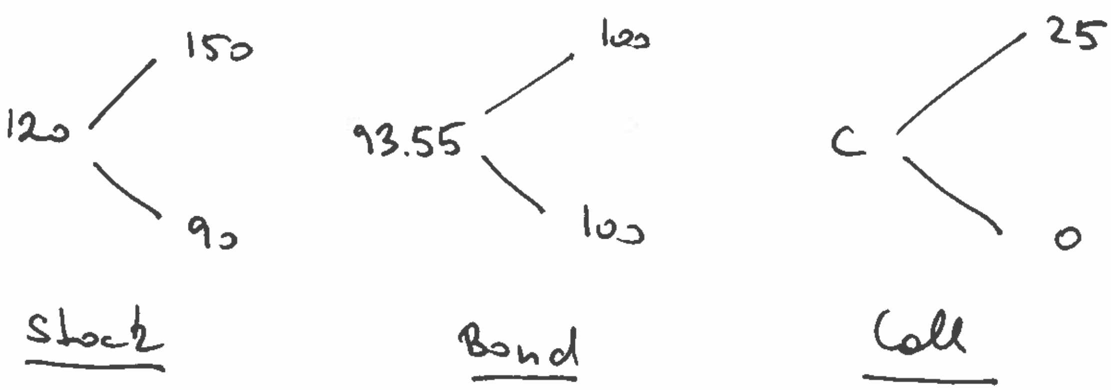

$$150 N_S + 100 N_B = 25$$

$$90 N_S + 100 N_B = 0.$$

$$N_S = \frac{25-0}{150-90} = 0.417 \qquad N_B = -\frac{90}{100} \times 0.417 = -0.375$$

$$\text{Call} = 0.417 \times 120 - 0.375 \times 93.55$$

$$= 14.96$$

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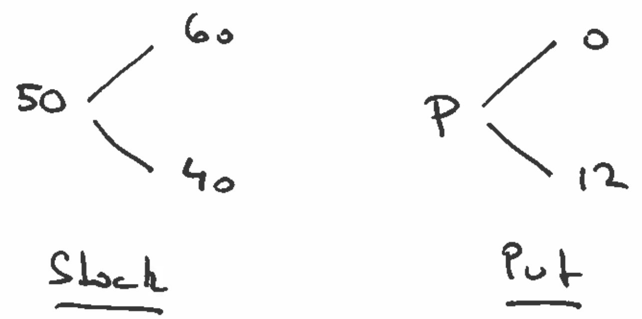

$$q = \frac{50\, e^{(0.08-0.04)\times 4/12} - 40}{60 - 40} = 0.534$$

$$\text{Put} = (0\, q + 12(1-q))\, e^{-0.08\times 4/12}$$

$$= 5.45$$

b. $$c - P = Se^{-\delta T} - Ke^{-rT}$$

$$c = P + Se^{-\delta T} - Ke^{-rT}$$

$$= 5.45 + 50e^{-0.04\times 4/12} - 52e^{-0.08\times 4/12}$$

$$= 4.16$$

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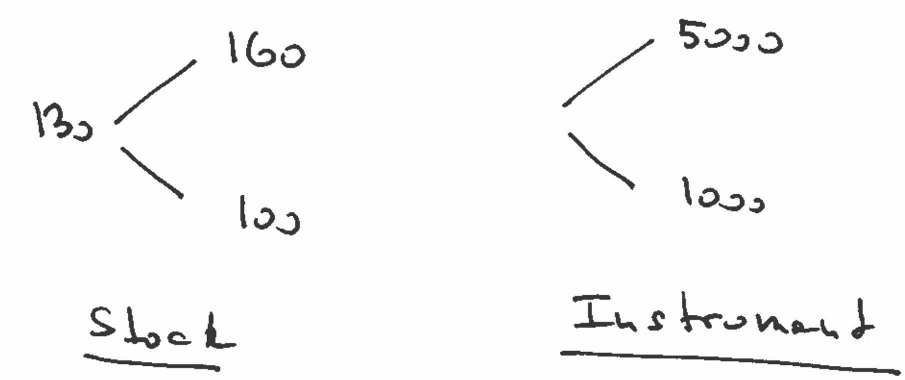

$$N_S = \frac{5000 - 1000}{160 - 100} e^{-0.10 \times 6/12} = 63.42$$

She needs (the bank) to buy approx. 63 shares to hedge the exposure.

Problem 4 (2 pts). A stock that pays a dividend yield of 4% per year costs \$200. Financial analysts at all major investment banks agree there is a 60% probability that the stock will trade for \$250 next year if the company succeeds in developing a novel AI algorithm. Otherwise, the stock price could fall to \$100. The risk-free rate is 4% per year and is expected to remain constant. What should be the price of a security that pays \$0 next year if the stock goes up and \$1,000 otherwise?

$$200 \begin{cases} 250 \\ 100 \end{cases} \qquad\qquad X \begin{cases} 0 \\ 1000 \end{cases}$$

Stock   Instrument

$$q = \frac{200 e^{0.04-0.04} - 100}{250-100} = \frac{100}{150} = \frac{2}{3}$$

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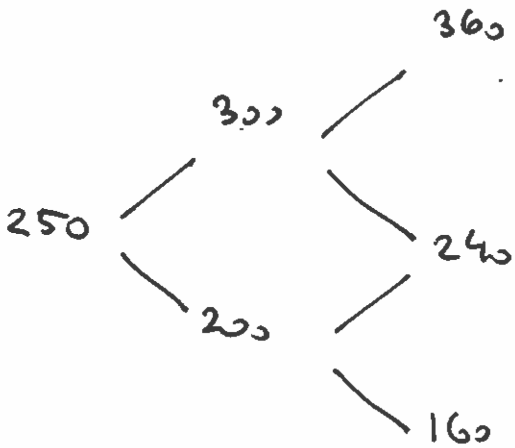
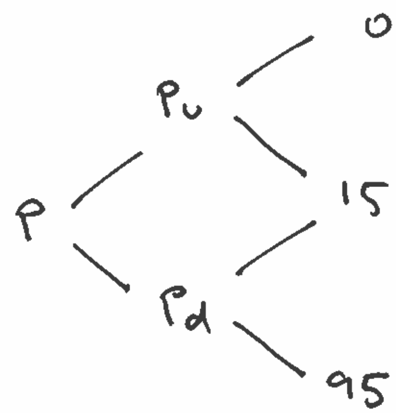


Stock



Put

$$q = \frac{250\, e^{0.10 \times 3/12} - 200}{300 - 200} = 0.563$$

If $S = 300$ in 3 months

Wait $= (0q + 15(1-q))\, e^{-0.10 \times 3/12} = 6.39$

exercise $= 0$

$P_u = 6.39$ ← we wait.

If $S = 200$ in 3 months

wait $= (15q + 95(1-q))\, e^{-0.10 \times 3/12} = 48.73$

exercise $= 255 - 200 = \underline{55}$ ← do it.

If $S = 250$ now

wait $= (6.39q + 55(1-q))\, e^{-0.10 \times 3/12} = \underline{\underline{26.95}}$ ← wait!

exercise $= 255 - 250 = 5$

$\boxed{\text{Put} = 26.95}$

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b. $$E \ln(Y_T) = \ln(2\,023.36) + \left(0.319 - \frac{1}{2} 0.75^2\right) \times \frac{6}{12}$$

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