Options, Futures and Derivative Securities | FIN 451
Time: 80 minutes | 20 points

Mock Midterm 1

Questions

Problem 1 (3 pts). Consider the following risk-free securities available to buy or sell to all investors in the market.

Security	Price (t=0)	Cash Flow (t=1)	Cash Flow (t=2)	Cash Flow (t=3)
A	?	10		
B	64		50	25
C	40			50
D	253	50	100	150

a. How can you synthesize D using A, B, and C?

$D = 5A + 2B + 2C$

b. What should be the no-arbitrage price of security A?

$5A = D - 2B - 2C$

$A = \frac{1}{5}(D - 2B - 2C)$

$P_A = \frac{1}{5}(253 - 2 \times (64 - 2 \times 40) = 9.$

c. If security A is trading at 8, is there an arbitrage opportunity? If so, explain how to exploit it. Be explicit about what you and what you sell, and in which quantities.

We know that

$$D = 5A + 2B + 2C$$

We can make D cheaper. So do it!

Buy	5A	-5×8
Buy	2 B	-2×64
Buy	2c	-2×40
Sell	D	$+253$
		$+5.$

Problem 2 (5 pts). The current GBP/USD exchange rate is \$1.26 per £. The interest rate in USD is 4% whereas the interest rate in GBP is 5%.

a. Compute the 2-year GBP/USD no-arbitrage forward price. Express your answer with four decimals.

$$F = 1.26\, e^{(0.04-0.05)\times 2} = 1.2351.$$

b. If the GBP/USD forward price is \$1.22 per £, is there an arbitrage opportunity? If so, explain how to exploit it by answering the questions below. Assume that you can buy or sell a maximum of £100 million forward.

   i. Do you buy or sell British pounds forward? How many? Buy £100 m forward
   ii. How many British pounds do you borrow or invest at the GBP interest rate? How much this corresponds in US dollars?
   iii. How many US dollars do you borrow or invest at the USD interest rate?

	$t=0$	$t=2$
i. Buy £100 m Fwd.	0	+£100 m = −\$122 m
iii. Deposit	−\$112.62 m	+ \$122 m
ii. Borrow	+ \$114.01 m	− £100 m
	+ \$1.39 m	0

$$\$112.62 = \$122\, e^{-0.04\times 2}$$

$$£100\; e^{-0.05\times 2} = £90.48 \xrightarrow{\times 1.26} \$114.01\text{ m}$$

c. Suppose that one year ago, you entered into a short forward to sell £100 million in two years at the forward price described in a. Now the forward has one year to go, the GBP/USD exchange rate is \$1.35 per £, and interest rates in USD and GBP have remained unchanged. If you decide to close your forward position, how much do you need to pay or get paid?

Current Forward Price $= 1.35\, e^{(0.04-0.05)\times 1} = 1.3366$

Value $= 100 \times (1.2357 - 1.3366)\, e^{-0.04\times 1}$

$= -\$9.75\text{m}$

You need to pay \$9.75m to get out of your position.

Problem 3 (2 pts). The CME crude oil futures contract is is defined over 1,000 barrels of light sweet crude oil. Say you deposit $50,000 in your margin account and buy one crude oil futures at $72.32. Complete the following table describing the evolution of your margin account.

Day	Futures Price	Gain/Loss	Margin Account
0	72.32	–	50,000.00
1	73.32	+1000	51,000.00
2	71.37	−1,950	49,050.00
3	71.29	−80	48,970

Problem 4 (2 pts). Stock XYZ current price is \$150, and calls expiring in six months with an exercise price of \$200 are selling at a premium of \$10 per share. With \$14,000 to invest, you are considering selling \$1,000 worth of calls and buying \$15,000 worth of XYZ stock. Compute the profit and the net rate of return of your portfolio six months from now if the price of XYZ stock at point in time is \$180.

	Payoff
Buy 100 shares	180×100
Sell 100 calls	0×100
	\$18,000

Payoff = \$18,000

Profit = \$18,000 − 14,000 = \$4,000.

Return = $\frac{4,000}{14,000}$ = 28.57%

Problem 5 (4 pts). An 8-month European call option on a non-dividend-paying stock is currently selling for \$5. The stock price is \$132, the strike price is \$130, and the risk-free interest rate is 4% per year. What opportunities could an arbitrageur exploit? Use the table below to indicate what you buy or sell (circle it in the table below), and compute the corresponding cash flows today and in eight months from the point of view of the arbitrageur.

Position	$t = 0$	$t = 8/12$	
		$S_t \leq 130$	$S_t > 130$
Buy / Sell Call	-5	0	$S_t - 130$
Buy / Sell Stock	$+132$	$-S_t$	$-S_t$
Borrow / Deposit	-126.58	$+130$	130
Total Cash Flow	$+0.42$	$130 - S_t > 0$	0

The lower bound for the call is

$$c \geq \max(\underbrace{132 - 130 e^{-0.04 \times 8/12}}_{5.42 > 5}, 0)$$

$$P = c - S + \underbrace{Ke^{-rT}}_{130 e^{-0.04 \times 8/12}} = 126.58$$

Problem 6 (1 pts). The price of a non-dividend paying stock is \$250. The risk-free rate is 5% per year with continuous compounding. Consider a European put option with strike price \$280 and maturity 1 year. What should be the price of the put if the volatility of the stock returns is extremely high?

Put price in this case is the upper bound.

$$P = Ke^{-rT} = 280\, e^{-0.05} = 266.34$$

Problem 7 (3pts). On 2/5/2025 at 10:28 AM CST, AMZN stock was trading for \$235.47. You got the following information on options with different strike prices and maturities written on AMZN.

	2025-03-07		2025-05-16		2025-08-15		2026-01-16	
Strike	Call	Put	Call	Put	Call	Put	Call	Put
210.0	29.97	2.12	34.99	5.80	40.60	9.38	48.80	13.40
215.0	25.70	2.97	31.00	7.25	37.05	10.95	46.16	15.05
220.0	21.25	4.14	27.58	8.72	34.40	12.55	43.00	15.60
225.0	18.20	5.67	25.24	10.50	30.70	12.75	39.53	18.84
230.0	14.90	7.50	21.40	12.50	28.00	16.05	36.50	21.20
235.0	11.40	9.65	18.90	14.80	25.35	18.65	33.90	23.29
240.0	9.05	11.60	16.49	17.30	22.80	21.55	31.36	25.70
245.0	7.22	15.40	14.10	19.70	21.05	24.30	29.05	26.10
250.0	5.52	18.50	12.08	22.70	18.90	27.15	26.80	28.94
255.0	4.05	20.90	10.40	23.20	16.90	29.45	24.60	34.03

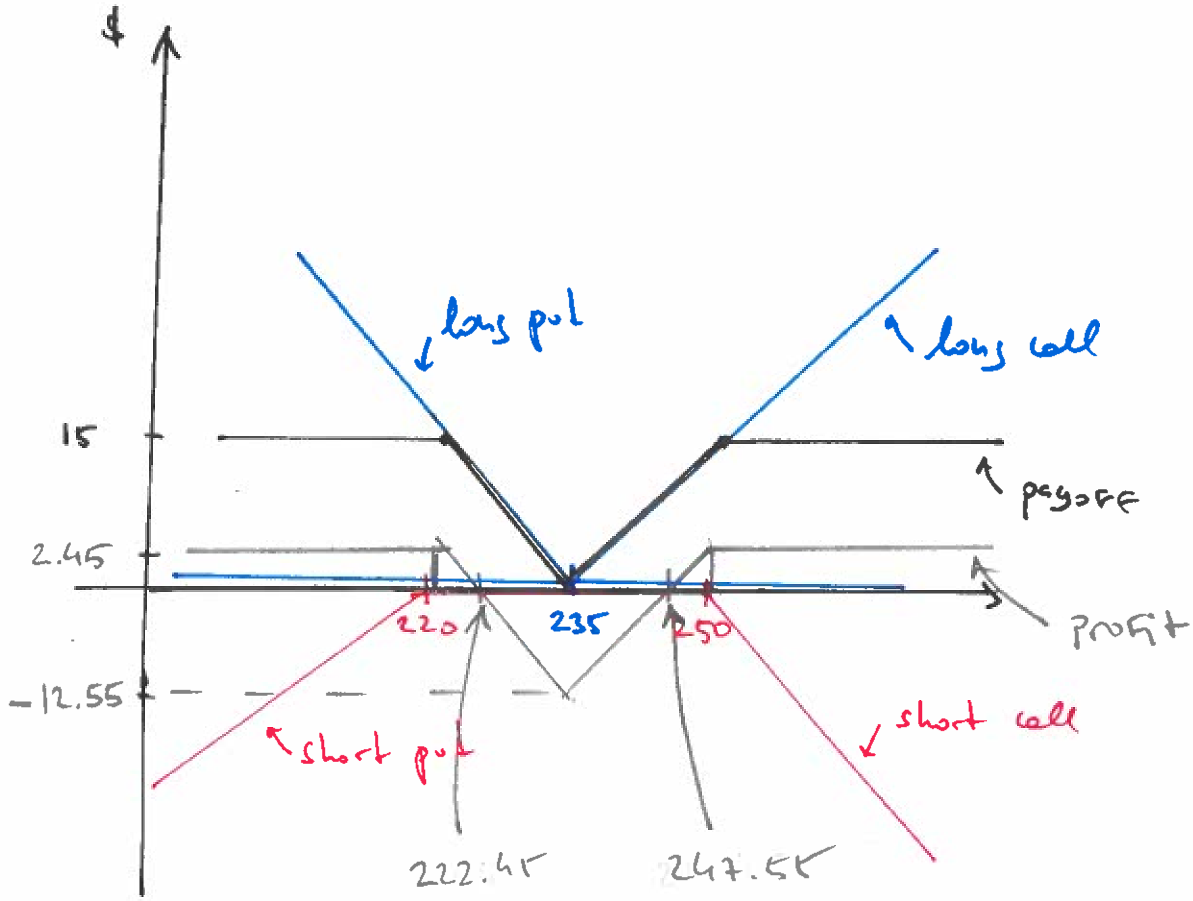
Draw the payoff and profit diagram as a function of the final stock price of a strategy in which you:

i. Buy a Aug 25 call with strike price 235.
ii. Buy a Aug 25 put with strike price 235.
iii. Sell a Aug 25 call with strike price 250.
iv. Sell a Aug 25 put with strike price 220.

In your diagram, indicate the cutoff prices that lead to a gain/loss.

Cost = 25.35 + 18.65 − 18.90 − 12.55

= \$12.55


$
long put
long call
15
payoff
2.45
profit
220
235
250
-12.55
short put
short call
222.45
247.55

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a. How can you synthesize D using A, B, and C?

$$D = 5A + 2B + 2C$$

b. What should be the no-arbitrage price of security A?

$$5A = D - 2B - 2C$$

$$A = \frac{1}{5}(D - 2B - 2C)$$

$$\text{Price}(A) = \frac{1}{5}(253 - 2 \times 64 - 2 \times 40) = 9$$

c. If security A is trading at 8, is there an arbitrage opportunity? If so, explain how to exploit it. Be explicit about what you and what you sell, and in which quantities.

$$D = 5A + 2B + 2C$$

Buy 5A	$-5 \times 8 = -40$
Buy 2B	$-2 \times 64 = -128$
Buy 2C	$-2 \times 40 = -80$
Sell D	$+253$
Total	$+5$.

Problem 2 (5 pts). The current GBP/USD exchange rate is \$1.26 per £. The interest rate in USD is 4% whereas the interest rate in GBP is 5%.

a. Compute the 2-year GBP/USD no-arbitrage forward price. Express your answer with four decimals.

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i. Do you buy or sell British pounds forward? How many?

ii. How many British pounds do you borrow or invest at the GBP interest rate? How much this corresponds in US dollars?

iii. How many US dollars do you borrow or invest at the USD interest rate? Deposit \$112.62m

	$t=0$	$t=2$
i. Buy £100m forward	0	~~£100m~~ − \$122m
iii. Deposit	− \$112.62m	+ ~~\$122m~~
ii. Borrow	+ \$114.01m	− ~~£100m~~
	+ \$1.39m.	0

$$\$122\, e^{-0.04\times 2} = \$112.62$$

$$£100\, e^{-0.05\times 2} = £90.48 = \$114.01\text{m}$$

$= 90.48 \times 1.26$

c. Suppose that one year ago, you entered into a short forward to sell £100 million in two years at the forward price described in a. Now the forward has one year to go, the GBP/USD exchange rate is $1.35 per £, and interest rates in USD and GBP have remained unchanged. If you decide to close your forward position, how much do you need to pay or get paid?

Current Forward Price $= 1.35 e^{(0.04-0.05)\times 1}$

$= \$1.3366$

USD

$$\text{Value} = £100m \times \underbrace{(1.2351 - 1.3366)}_{\$/£} e^{-0.04\times 1}$$

$= -\$9.75m$

You would have to pay $9.75m to close the position.

Problem 3 (2 pts). The CME crude oil futures contract is is defined over 1,000 barrels of light sweet crude oil. Say you deposit \$50,000 in your margin account and buy one crude oil futures at \$72.32. per barrel.
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	Payoff
Buy 100 shares	18,000
Sell 100 calls	0
Total	18,000

Profit = 18,000 − 14,000 = 4,000

Return = $\frac{4,000}{14,000}$ = 28.57%

Problem 5 (4 pts). An 8-month European call option on a non-dividend-paying stock is currently selling for $5. The stock price is $132, the strike price is $130, and the risk-free interest rate is 4% per year. What opportunities could an arbitrageur exploit? Use the table below to indicate what you buy or sell (circle it in the table below), and compute the corresponding cash flows today and in eight months from the point of view of the arbitrageur.

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Buy / Sell Call	-5	0	$\cancel{S_t} - \cancel{130}$
Buy / Sell Stock	$+132$	$-S_t$	$-\cancel{S_t}$
Borrow / Deposit	-126.58	130	$\cancel{130}$
Total Cash Flow	$+0.42$	$130 - S_t$	0

$$c \geqslant \max\left(132 - 130\, e^{-0.04 \times 8/12},\ 0\right) = 5.42$$

$$P = \underline{c} - S + \underbrace{Ke^{-rT}}_{130\, e^{-0.04 \times 8/12} \,=\, 126.58}$$

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The price this put should have is the upper bound of the put premium. That is,

$$280 e^{-0.05 \times 1} = 266.34.$$

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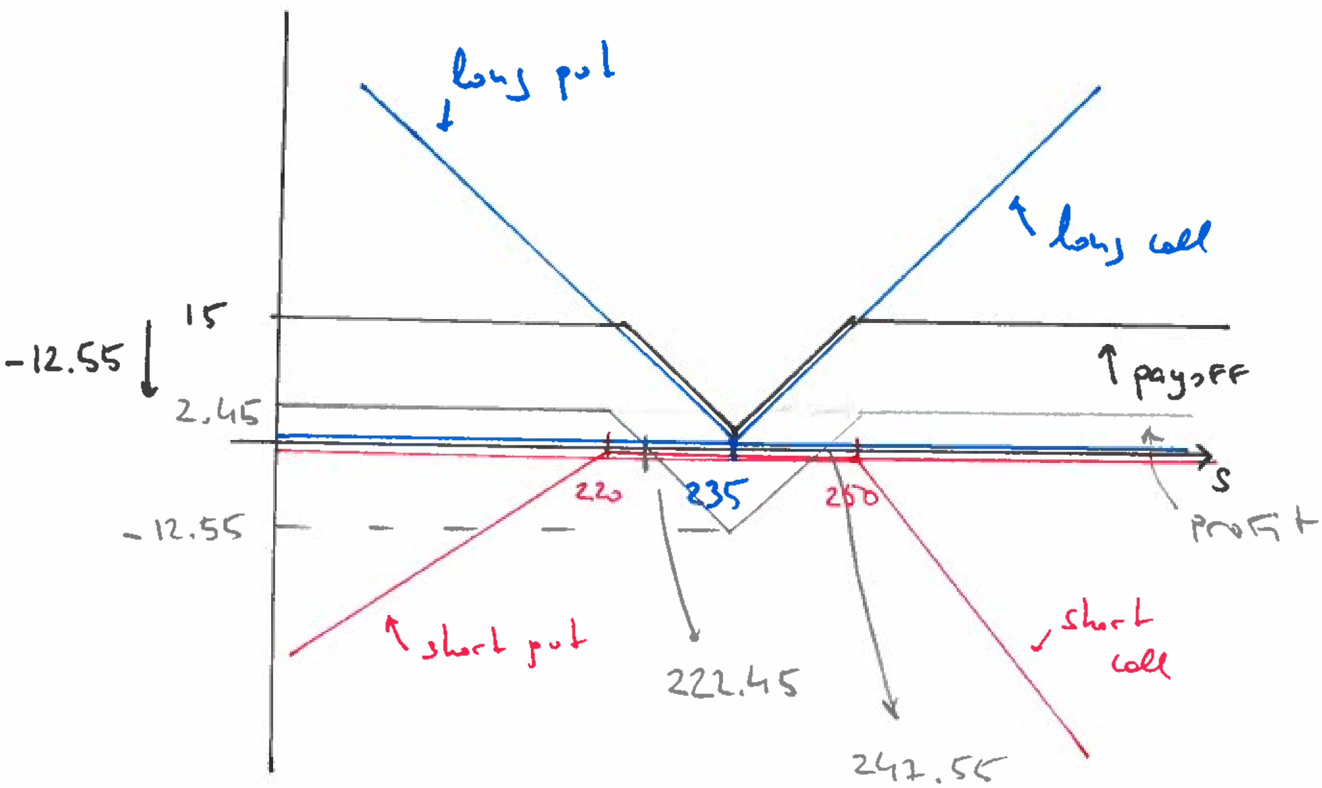
long put
long call
15
-12.55
2.45
payoff
220
235
250
S
-12.55
profit
short put
short call
222.45
247.55