The Impact of Dividends

Assets Paying Cash Dividends

Put-Call Parity with Dividends

For European options written on stocks paying cash dividends, the put-call parity is modified as follows:

$$C - P = S - D - Ke^{-rT}$$

where D is the present value of dividends paid during the life of the option.

In order to derive this expression we will proceed as before by trying to build a covered call in two different ways. Suppose that we did the same as in the previous lecture.

Strategy A: Long stock and short call

$$\text{Cost} = S - C$$

$$\text{Payoff} = \begin{cases} S_T + FV(D) & \text{if } S_T \leq K \\ K + FV(D) & \text{if } S_T > K \end{cases}$$

Strategy B: Long bond and short put

$$\text{Cost} = Ke^{-rT} - P$$

$$\text{Payoff} = \begin{cases} S_T & \text{if } S_T \leq K \\ K & \text{if } S_T > K \end{cases}$$

We can see that both strategies no longer have the same payoff at maturity because the stock pays dividends. Let us then adjust the first strategy to account for the fact that the stock pays dividends.

Strategy A: Long stock, borrow D and short call

$$\text{Cost} = S - D - C$$
$$\text{Payoff} = \begin{cases} S_T & \text{if } S_T \leq K \\ K & \text{if } S_T > K \end{cases}$$

Strategy B: Long bond and short put

$$\text{Cost} = Ke^{-rT} - P$$
$$\text{Payoff} = \begin{cases} S_T & \text{if } S_T \leq K \\ K & \text{if } S_T > K \end{cases}$$

Note that in A we use the dividends, reinvested at r, to repay the loan and generate the same payoff as in B.

Example 1. Suppose that $S = 110$, $r = 5\%$, $K = 110$, $T = 9$ months, and $C = 13.30$. The stock is expected to pay dividends of \$2 in six months, and \$2.5 in one year. What should be the no-arbitrage price of a European put with the same strike and maturity as the European call?

In order to use put-call parity to find the price of the European put, we need to compute the present value of the dividends that will be paid during the life of the option. Hence, we only consider the dividend that will be paid at $t = 6/12$ but not the dividend that will be paid at $t = 1$:

$$D = 2e^{-0.05\times 6/12} = 1.95.$$

Then, according to put-call parity we should have that a European put with the same strike and maturity as the call should cost:

$$P = 13.30 - 110 + 1.95 + 110e^{-0.05\times 9/12} = 11.20.$$

The put should then trade for \$11.20. □

Example 2. What if in the previous example everything stays the same, but you find that the put trades for \$11?

	$T=0$	$T=6/12$	$T=9/12$	
Transactions			$S_T \leq 110$	$S_T > 110$
Long put	-11.00	0	$110 - S_T$	0
Short call	13.30	0	0	$-(S_T - 110)$
Long stock	-110.00	2	S_T	S_T
Borrow D	1.95	-2	0	0
Short bond	105.95	0	-110	-110
Total	0.20	0	0	0

Then we have an arbitrage opportunity since the put is relatively cheap compared to what it should trade. Hence, we should buy the put and sell the synthetic put. Note that the stock will pay a dividend of \$2 in six months that can be used to pay the loan at that time. □

Lower Bound on European Options with Dividends

With dividends, we modify the lower bounds on European call and put options as follows:

$$C \geq \max(S - D - Ke^{-rT}, 0)$$
$$P \geq \max(Ke^{-rT} - S + D, 0)$$

Example 3. Suppose that you have $S = 110$, $r = 5\%$, $K = 110$, and $T = 9$ months. The stock is expected to pay a dividend of \$2 in six months, and of \$2.5 in 1 year. The previous result implies that:

$$C \geq \max(110 - 2e^{-0.05\times 6/12} - 110e^{-0.05\times 0.75}, 0) = \max(2.10, 0) = 2.10$$

Hence, no matter how low the volatility is on this European call option with strike \$110 and maturity 9 months, its premium must be higher than \$2.10. □

Assets Paying a Dividend Yield

The Dividend Yield

There are many assets that pay dividends continuously, like a currency, or that can be modeled as such, like a stock index. In these cases it is convenient to model dividends as a percentage yield paid over time. We will denote the continuously-compounded dividend yield by δ.

The asset S then pays every instant t a dividend of $\delta S_t \Delta t$. Therefore, the dividend yield can be seen as the units of the asset growing over time at the rate q. In practice, this is the approach used to model options on stock indices and currencies, although some practitioners also use it to model individual stocks as well.

Put-Call Parity and European Options Bounds

> Put-Call Parity with a Dividend Yield
>
> For European options written on assets paying a dividend yield, the put-call parity is modified as follows:
> $$C - P = Se^{-\delta T} - Ke^{-rT}$$
> where δ is the dividend yield paid continuously by the asset during the life of the option.

In order to derive this expression we will proceed as before by trying to build a covered call in two different ways.

Strategy A: Long $e^{-\delta T}$ units of the asset and short call

$$\text{Cost} = Se^{-\delta T} - C$$
$$\text{Payoff} = \begin{cases} S_T & \text{if } S_T \leq K \\ K & \text{if } S_T > K \end{cases}$$

Strategy B: Long bond and short put

$$\text{Cost} = Ke^{-rT} - P$$

$$\text{Payoff} = \begin{cases} S_T & \text{if } S_T \leq K \\ K & \text{if } S_T > K \end{cases}$$

Note that in A the asset grows at the rate δ, so the total number of units of the asset at maturity is $e^{-\delta T}e^{\delta T} = 1$.

Example 4. Suppose that $S = 110$, $r = 5\%$, $\delta = 3\%$, $K = 110$, $T = 9$ months, and $C = 13.30$. What should be the no-arbitrage price of a European put with the same strike and maturity as the European call?

According to put-call parity we should have that a European put with the same strike and maturity as the call should cost:

$$P = 13.30 - 110e^{-0.03\times 9/12} + 110e^{-0.05\times 9/12} = 11.70.$$

□

Example 5. What if in the previous example everything stays the same, but you find that the put trades for $11?

	$T = 0$	$T = 9/12$	
Transactions		$S_T \leq 110$	$S_T > 110$
Long put	–11.00	$110 - S_T$	0
Short call	13.30	0	$-(S_T - 110)$
Long $e^{-\delta T}$ units of asset	–107.55	S_T	S_T
Short bond	105.95	–110	–110
Total	0.70	0	0

Then we have an arbitrage opportunity since the put is relatively cheap compared to what it should trade. Hence, we should buy the put and sell the synthetic put. Note that because the asset pays a dividend yield we need to purchase slightly less of it today in order to get one unit if the asset in nine months. □

Lower Bounds of European Options with Dividend Yields

If the asset pays a dividend yield, we modify the lower bounds on European call and put options as follows:

$$C \geq \max(Se^{-\delta T} - Ke^{-rT}, 0)$$
$$P \geq \max(Ke^{-rT} - Se^{-\delta T}, 0)$$

Example 6. Suppose that you have $S = 110$, $r = 5\%$, $\delta = 3\%$, $K = 110$, and $T = 9$ months. The previous result implies that:

$$C \geq \max(110e^{-0.03\times 9/12} - 110e^{-0.05\times 9/12}, 0) = \max(1.60, 0) = 1.60$$

Hence, no matter how low the volatility is on this European call option with strike $110 and maturity 9 months, its premium must be higher than $1.60. □

Since the payoffs of strategies A and B described above are positive, the cost of both strategies must be positive.

Upper Bounds of European Options with Dividend Yields

If the asset pays a dividend yield, the upper bounds on European call and put options are as follows:

$$C \leq Se^{-\delta T}$$
$$P \leq Ke^{-rT}$$

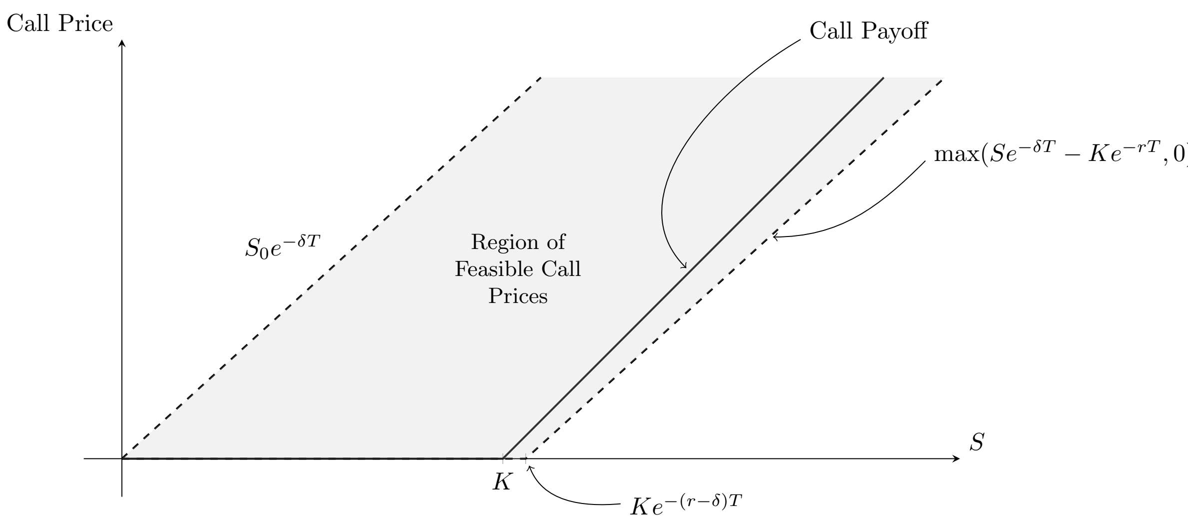
Example 7. Suppose that you have $S = 110$, $r = 5\%$, $\delta = 3\%$, $K = 110$, and $T = 9$ months. The previous result implies that:

$$C \leq 110e^{-0.03\times 9/12} = 107.55$$

Hence, no matter how high the volatility is on this European call option with strike \$110 and maturity 9 months, its premium must be less than \$107.55. □

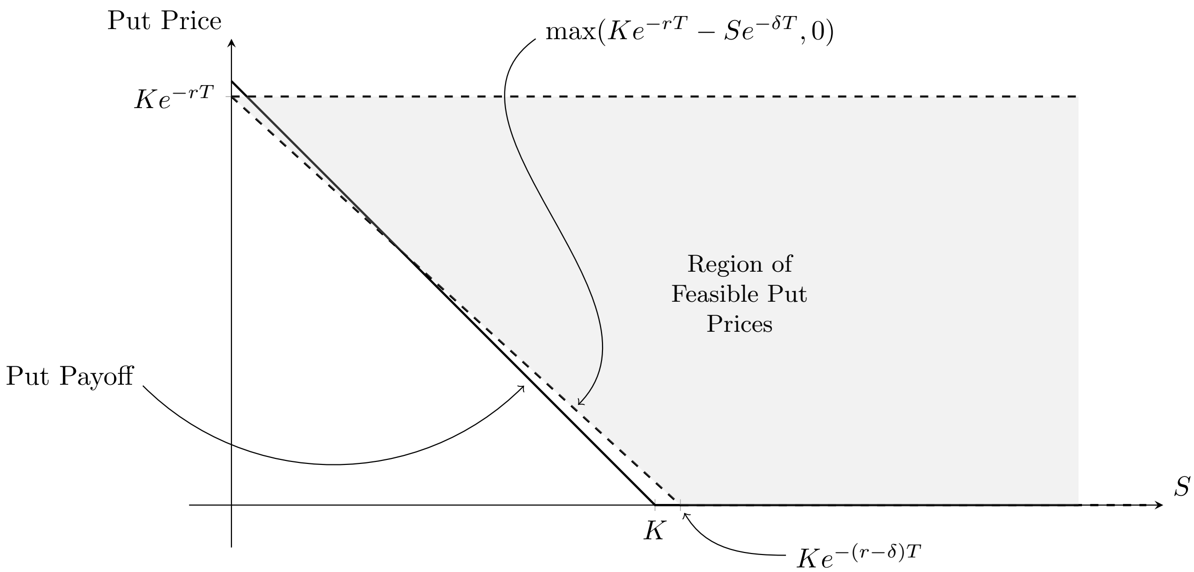
Feasible Prices for European Call Options

The graph describes the region of feasible prices for European call options written on an asset that pays a positive dividend yield such that $\delta > r$.



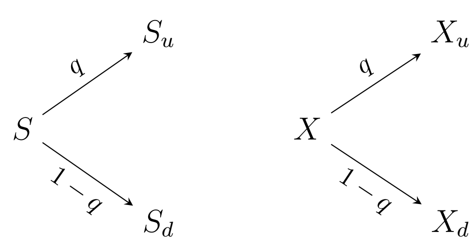
Feasible Prices for European Put Options

The graph describes the region of feasible prices for European put options written on an asset that pays a positive dividend yield such that $\delta > r$.



Binomial Pricing

Pricing options when the asset pays a dividend yield requires to adjust the risk-neutral probabilities accordingly. Say that over the next period Δt the asset price can go up to $S_u = Su$, or down to $S_d = Sd$, and that we want to price a derivative X that pays either X^u or X^d in each state, respectively.



The risk-neutral probability of an up-move in this case is given by:

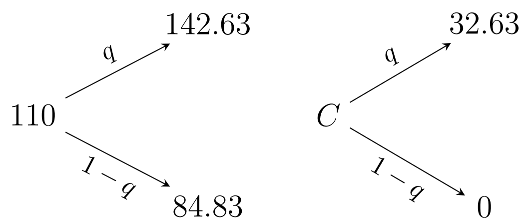
$$q = \frac{e^{(r-\delta)\Delta t} - d}{u - d}$$

The price of the derivative is then:

$$X = (qX_u + (1-q)X_d)e^{-r\Delta t}$$

Note that we can make this model consistent with the Black-Scholes model by choosing $u = e^{\sigma\sqrt{\Delta t}}$ and $d = 1/u$, where σ represents the annualized volatility of the asset returns.

Example 8. Suppose that $S = 110$, $r = 5\%$, $\delta = 3\%$, $\sigma = 30\%$, $K = 110$, $T =$ 9 months. Using a one-period binomial tree, let's compute the no-arbitrage price of a European call option. The binomial trees for the asset and the call are as follows:



$$q = \frac{110e^{(0.05-0.03)\times 9/12} - 84.83}{142.63 - 84.83} = 0.4642$$
$$C = (32.63q + 0(1-q))e^{-0.05\times 9/12} = 14.59$$

□

Practice Problems

Solutions to all problems can be found at lorenzonaranjo.com/fin451.

Problem 1. A four-month European call option on a dividend-paying stock is currently selling for \$5. The stock price is \$64, the strike price is \$60, and a dividend of \$0.80 is expected in one month. The risk-free interest rate is 12% per annum for all maturities. What opportunities are there for an arbitrageur?

Problem 2. The price of a European call that expires in six months and has a strike price of \$30 is \$2. The underlying stock price is \$29, and a dividend of \$0.50 is expected in two months and again in five months. The term structure is flat, with all risk-free interest rates being 10%.

a. What is the price of a European put option that expires in six months and has a strike price of $30?
b. Explain carefully the arbitrage opportunities if the European put price was $3 instead of the price computed in 1.

Problem 3. A European call option and put option on a stock both have a strike price of $20 and an expiration date in three months. Both sell for $3. The risk-free interest rate is 10% per annum, the current stock price is $19, and a $1 dividend is expected in one month. Identify the arbitrage opportunity open to a trader.

Problem 4. Suppose that $S = 110$, $r = 4\%$, $\delta = 8\%$, and $\sigma = 40\%$. Using a two-period binomial tree, compute the no-arbitrage price of a European put option with strike $112 and maturity 8 months.

Problem 5. A stock that pays a dividend yield of 8% per year costs $50. Analysts estimate that there is a 50% chance that the stock trades for $60 next year if the company succeeds in developing an important vaccine. Otherwise, the stock could fall to $40. The risk-free rate is 5% per year with continuous compounding. What should be the price of a security that pays $100 next year if the stock goes up, and $0 otherwise?

Problem 6. The current price of a stock is $200. Over the next year, it is expected to go up or down by 11% or 15%, respectively. The stock pays a dividend yield of 6% per year and the risk-free rate is 7% per year with continuous compounding. A market-maker of an important investment bank just sold 100 at-the-money European call options (i.e. one contract) expiring in one year to an important client. How many shares of the stock does she need to buy in order to hedge her exposure?

Problem 7. Consider an asset that trades for $200. Over the next six months, analysts expect that it could go up to $213 or down to $183. The asset pays a dividend yield of 7% per year. Compute the price of an at-the-money European call option expiring in six months. Assume that the risk-free rate is 9% per year with continuous compounding.

Problem 8. The current price of a stock is \$100. Every three months, it is expected to go up or down by 15% or 11%, respectively. The stock pays a dividend yield of 7% per year and the risk-free rate is 8% per year with continuous compounding. Compute the price of a European call option with strike price \$98 and maturity six months written on the stock.

Problem 9. Consider an asset that trades for \$200. Over the next six months, analysts expect that it could go up to \$212 or down to \$186. The asset pays a dividend yield of 6% per year. Compute the price of an at-the-money European put option expiring in six months. Assume that the risk-free rate is 7% per year with continuous compounding.