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Futures Markets

Definitions

Futures trade in organized exchanges such as the Chicago Mercantile Exchange (CME) or the Chicago Board Options Exchange (CBOE). Futures contracts are written on a wide variety of asset classes such as physical commodities, equity indices, foreign currencies, interest rates and even abstract quantities such as volatility.

Traditionally, all futures used to trade in physical trading floors organized into segmented areas called pits. The idea was for traders and floor brokers to interact directly with each other in face-to-face transactions, in a system commonly known as *open-outcry*. With the advent of technology and later on the COVID-19 pandemic, almost all futures trading today occurs electronically.

Example 1. These are examples of some futures contracts:

- Buy 100 oz. of gold @ $1400 per oz. in December
- Sell £62,500 at $1.2620 per £ in June
- Sell 1,000 barrels of oil at $70 per barrel in July

Electronic trading is usually implemented by recording bids and offers in what is called an electronic *limit-order book*. Limit orders, if not executed immediately either because the bid is lower than the best offer price, or because the offer is higher than the best bid, are recorded in the book. These orders can be cancelled at no cost, otherwise they will stay active usually until the end of that day's trading session. Market orders on the other hand will execute immediately at the best possible bid or offer available at order received time.

Unlike stocks, futures exchanges have longer trading hours. For example, the market hours for E-mini S&P 500 (ES), which is traded online, is Sunday through Friday 5 p.m. to 4 p.m. Central Time (CT).

Settlement and Maturity

The settlement of a futures contract happens if the trader keeps the position open until maturity. For many commodity contracts, the settlement procedure involves the physical delivery of the underlying asset. Therefore, the contract specifications must clearly specify the size, quality and physical delivery options to receive or deliver the commodity. [1]

> Settlement of Soybean Futures
>
> The following paragraph is from the CME rulebook on soybean futures:
> *Each futures contract shall be for 5,000 bushels of No. 2 yellow soybeans at par, No. 1 yellow soybeans at 6 cents per bushel over contract price, or No. 3 yellow soybeans at 6 cents per bushel under contract price provided that all factors equal U.S. No. 2 or better except for foreign material (refer to Rule 11104.). Every delivery of soybeans may be made up of the authorized grades for shipment from eligible regular facilities provided that no lot delivered shall contain less than 5,000 bushels of any one grade from any one shipping station.*
> The complete rulebook for soybean futures can be found at the CME website.

For equity index and interest rates futures, the delivery takes place as a cash settlement. The value of the contract is computed as a dollar amount times the value of an reference index.

All futures contracts have an expiration date, and for a given underlying asset there are usually several expirations trading at any given point in time.

[1] The delivery size of the contract determines the notional value at maturity, which in turns determine the minimum margin requirement.

E-mini S&P 500 Futures Expirations

E-mini S&P 500 futures have quarterly contracts (Mar, Jun, Sep, Dec) listed for 9 consecutive quarters and 3 additional December contract months. As of 5/24/2022, the table below presents the available maturities of these contracts.

Month	Ticker	First Trade	Last Trade	Settlement
Jun-22	ESM22	19-Mar-21	17-Jun-22	17-Jun-22
Sep-22	ESU22	07-Jun-21	16-Sep-22	16-Sep-22
Dec-22	ESZ22	07-Jun-21	16-Dec-22	16-Dec-22
Mar-23	ESH23	07-Jun-21	17-Mar-23	17-Mar-23
Jun-23	ESM23	07-Jun-21	16-Jun-23	16-Jun-23
Sep-23	ESU23	18-Jun-21	15-Sep-23	15-Sep-23
Dec-23	ESZ23	07-Jun-21	15-Dec-23	15-Dec-23
Mar-24	ESH24	17-Dec-21	15-Mar-24	15-Mar-24
Jun-24	ESM24	18-Mar-22	21-Jun-24	21-Jun-24
Dec-24	ESZ24	07-Jun-21	20-Dec-24	20-Dec-24
Dec-25	ESZ25	07-Jun-21	19-Dec-25	19-Dec-25
Dec-26	ESZ26	17-Sep-21	18-Dec-26	18-Dec-26

Futures vs. Spot Prices

Each of these contracts trade continuously during the exchange trading hours. Each day, for margin requirement purposes, the exchange determines the *settlement price* of the contract. The settlement price is what is usually reported as the price of the contract for that day. Figure 1 plots the evolution of the March 2025 Soybean settlement futures price.

We define the *spot price* of the underlying asset as as the closest-to-maturity futures price. For many commodities, the spot price is close but not the same as the cash price. Indeed, the delivery method of a futures contract might be different from the typical delivery method of

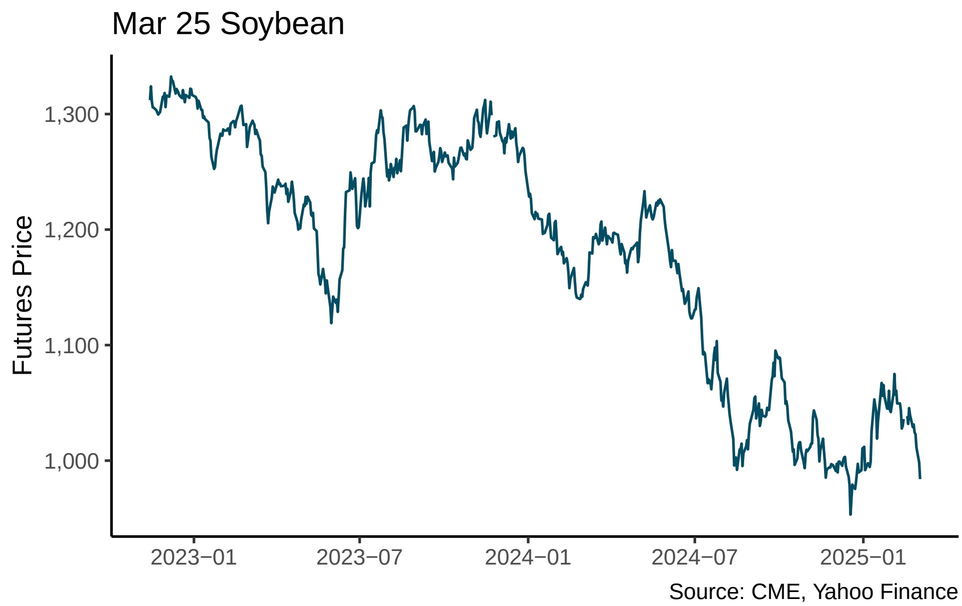

Figure 1: The figure shows the evolution of the Mar 25 Soybean futures contract over time.

the physical commodity. What makes a very short-maturity futures interesting for us is that it is relatively easily to sell a futures contract, whereas it is in general hard to short-sell a physical commodity. As a consequence, when analyzing commodity futures we will usually consider the shortest-to-delivery futures as our underlying asset and not necessarily the physical commodity.

More formally, if we denote by $F(t,T)$ the futures price at time t of a contract expiring at time T, the spot price is defined as:

$$S_t = F(t,t)$$

The figure below plots the evolution of the cotton spot price and the March 2025 futures contract on cotton since 2018.

We can see from the figure that the futures price converges over time to the spot price.

Exchanges also report the total number of contracts open which is usually called the *open interest*. Remember that for any long position there has to be a corresponding short position.

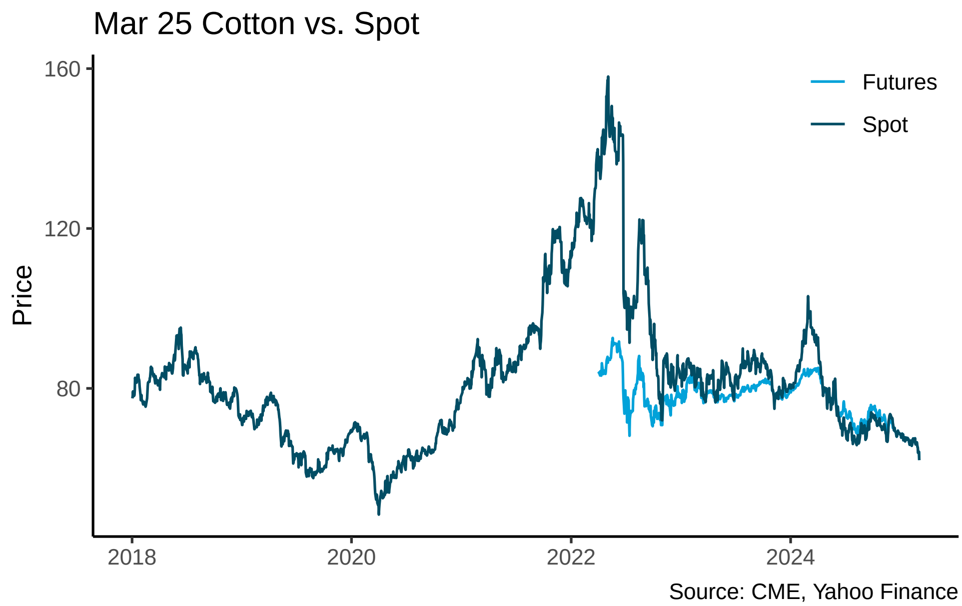

Figure 2: The figure shows the evolution of the March 25 cotton futures and the spot price over time.

Therefore, the open interest measures the total number of long positions, or equivalently the total number of short positions open at any given point in time.

Figure 3 displays the evolution of the open interest for all soybean, gold and oil futures starting Jun-2006.

It is important to note that a large open interest could reflect strong demand for short positions, long positions, or both.

The Commodity Futures Trading Commission (CFTC) publishes detailed weekly information about open interest for all futures in their Commitments of Traders (COT) Reports.

Short Selling

There are cases when we can relate the cash or physical commodity price with the spot price of the underlying asset. For this to happen, we must be able to freely purchase the physical

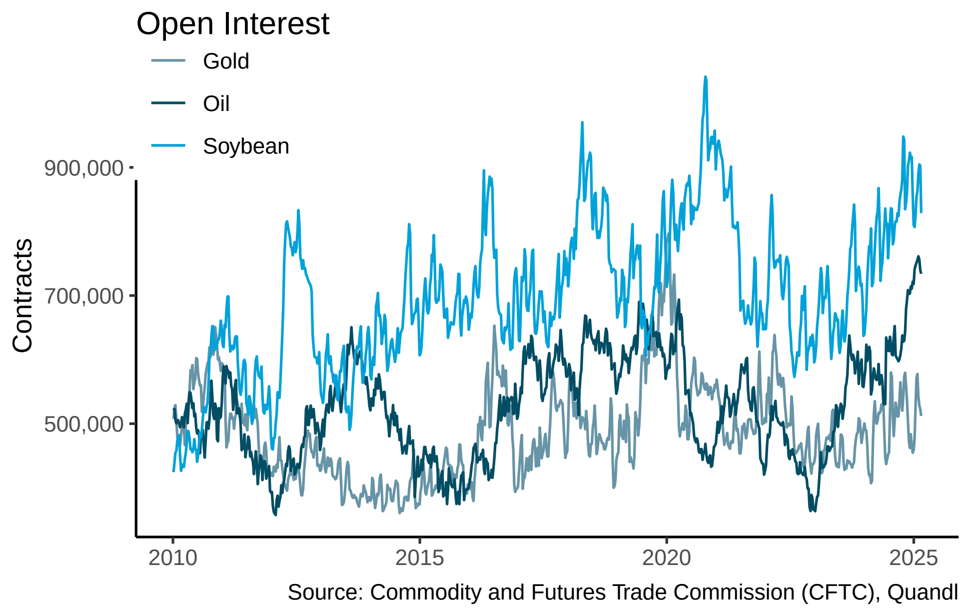

Figure 3: The figure shoes the evolution of the open interest for different futures over time.

commodity in both positive and negative quantities so we can arbitrage away any price inconsistencies between cash and spot markets. In other words, we must be able to *short-sell* the asset. This would be the case of stocks, for example.

Example 2 (Short selling a stock)**.** You short 100 shares when the price is \$100 and close out the short position three months later when the price is \$90. At the end of the three months and just before closing the position, a dividend of \$3 per share is paid. In this case your profit is

$$100 \times (100 - 90 - 3) = \$700.$$

If on the other hand you would have bought 100 shares, you would have had a profit of

$$100 \times (90 + 3 - 100) = -\$700.$$

Note that when you buy the stock you receive the dividends whereas when you short sell the stock you must pay those dividends back.

Short selling involves selling securities you do not own. Your broker borrows the securities from another client and sells them in the market in the usual way. At some stage, you must buy the securities back so they can be replaced in the account of the client. You must pay dividends and other benefits that the owner of the securities receives. There may be a small fee for borrowing the securities.

However, there are many futures contracts written on underlying assets that are difficult or impossible to short-sell. Think for example about commodities. It might be hard or impossible to short-sell oil or physical gold. In these cases we will resort to buying or selling the shortest futures contract in order to establish a relationship between the spot and the futures price.

Index Futures

Futures contracts written on stock market indices are called *index futures*. There are index futures written on the S&P 500, Nasdaq, Dax 30, CAC 40, Stoxx 50, among many others. In general, index futures are very liquid and provide good trading opportunities for both speculators and hedgers.

Example 3. Consider an index tracking a portfolio of stocks that pays a dividend yield of 3% per year with continuous compounding. The index is currently at 4,300. The risk-free rate for all maturities is 1% per year continuously-compounded. The 6-month futures price can be computed as

$$F = 4300e^{(0.01-0.03)\times 6/12} = 4257.21.$$

Note that because the dividend yield is higher than the risk-free rate, the futures price is less than the current spot price.

Because the tradable underlying asset is the basket of stocks that define the index, index futures are cash settled. Otherwise, it would be very cumbersome and most likely impossible to deliver a basket containing all the stocks in the index in the right proportions. Cash settlement means that if the contract reaches maturity, the contract is just marked-to-market against the value of the index. As a matter of fact, it is a pretty convenient way to settle to the contract.

In Example 3 we compute the no-arbitrage futures price of the index. When the observed futures price deviates from this relationship, arbitrageurs can then try to exploit this difference in their advantage in what is called *index-arbitrage*. Generally speaking, if F denotes the observed futures price, we have that:

- If $F < Se^{(r-\delta)T}$ then you should buy the futures, sell $e^{-\delta T}$ units of the index and invest Fe^{-rT} dollars in a risk-free money-market account.
- If $F > Se^{(r-\delta)T}$ then you should sell the futures, buy $e^{-\delta T}$ units of the index and borrow Fe^{-rT} dollars.

Note that in both scenarios the arbitrageur will make risk-free money today while being completely hedged when the futures reaches maturity.

If in Example 3 the 6-month futures is trading at 4,300, an arbitrageur could engage in the following transactions:

	$T=0$	$T=6/12$
Short Futures	0	$4300-S_T$
Buy $Se^{\delta T}$ units of the Index	-4235.98	S_T
Borrow Fe^{-rT}	4278.55	-4300
Total	42.57	0

The strategy generates a risk-free cash flow today of $42.57 per futures sold with no initial investment required. An arbitrageur could easily sell, just to give an arbitrary number, one million index futures, hedge accordingly and pocket $42.57 million.

This shows that in equilibrium the futures price cannot deviate much from its no-arbitrage price.

Commodity Futures

There are many futures written on commodities such as crude oil, copper, gold, soybean, among others. It is important to note that for commodities, the dividend yield corresponds to the net benefit accruing to the owner of the physical commodity but not to the buyer of a futures contract and is called the *convenience yield*.

The convenience yield should take into consideration the gross benefits of owning the physical commodity, such as the ability to profit from temporary shortages, but also storage costs. The difference between the risk-free rate and the convenience yield is usually called the *cost-of-carry*.

Example 4. Suppose that the spot price of oil is \$95 per barrel, the 1-year US\$ interest rate is 5% per year with continuous compounding and the convenience yield is 2% per year. The 1-year oil futures price is

$$F = 95e^{0.05-0.02} = \$97.89.$$

Futures vs. Forward Contracts

Even though futures and forward contracts are very similar, in practice there are important differences between these two instruments:

1. Futures contracts trade in exchanges such as the Chicago Mercantile Exchange (CME) whereas forwards trade the over-the-counter (OTC) where traders working for buy-side companies such as fund managers and corporate treasurers contact sell-side investors such as large international banks directly.
2. Futures exchanges standardize the terms of the contract such as expiration dates, notional amount, delivery method, and quality, among others. Forwards can be negotiated so that to fit specific needs of a client.
3. Futures exchanges require traders to keep a margin account which consists in cash or marketable securities deposited by an investor with his/her broker. The margin account balance is adjusted daily to account for daily gains or losses, and must always be above a certain minimum. Margins minimize potential losses that might occur because of a default event. Forward contracts, in general, are settled in full at expiration.

Notwithstanding these important differences, forward and futures prices with the same maturity are usually assumed to be equal. Indeed, when interest rates are deterministic (or uncorrelated with the underlying asset), futures and forward prices are the same. Given that this is the assumption we use in many chapters of this book to price options and futures, in what follows we will refer to either a futures or forward contract *interchangeably* unless stated otherwise.

It is important to note, however, that when interest rates are stochastic, futures and forward prices are in theory different:

- A strong positive correlation between interest rates and the asset price implies the futures price is higher than the forward price, as would be the case for Eurodollar futures (soon to be replaced by SOFR futures).
- A strong negative correlation between risk-free rates and the underlying asset implies the reverse. For interest rate futures, it is common to adjust the relevant forward rate in order to derive the futures rate. Such a modification is usually called a *convexity adjustment*.

Futures Prices vs. Expected Future Spot Prices

Many practitioners and academics have suggested that we could use futures or forward prices to forecast future spot prices. Intuitively, since futures prices determine the price at which an asset can be bought or sold in the future suggests that buyers and sellers should somehow use their forecasts when trading the derivative.

Whether or not futures prices are the best unbiased predictor of expected future prices depends on whether investors command a risk-premium to hold the asset. In stock markets the risk-premium is usually positive since stocks usually comove positively with the market portfolio. For currencies and commodities, though, the risk-premium could be positive, negative or zero depending on which side puts the hedging pressure.

For example, consider a commodity where producers want to hedge their production. If there are no consumers that want to hedge their consumption, then long futures position will have to be taken by speculators. If hedgers are risk-averse, they will then be willing to sell for less than the expected future price in order to unwind their risk-exposure.

Specifically, suppose that μ is the expected return required by investors and denote by F the futures price expiring at time T.

Imagine that you want to compute the value of 1 unit of the asset paid at time T. You could do this in two different ways. Either you discount the expected value of the asset at μ, or you hedge your exposure by selling a futures, which provides a certain cash flow F that can then be discounted at the risk-free rate. Therefore,

$$Fe^{-rT} = \mathsf{E}(S_T)e^{-\mu T},$$

or

$$F = \mathsf{E}(S_T)e^{(r-\mu)T}.$$

The previous expression shows that futures prices are unbiased estimators of future prices only when there is no systematic risk priced into the asset. More generally, futures prices will under- or over-estimate expected future prices depending on the sign of the systematic risk, as shown in the table below.

Type of Risk	Expected Return	Futures vs. Expected Price
No Systematic Risk	$\mu = r$	$F = \mathsf{E}(S_T)$
Positive Systematic Risk	$\mu > r$	$F < \mathsf{E}(S_T)$
Negative Systematic Risk	$\mu < r$	$F > \mathsf{E}(S_T)$

To determine whether there is systematic risk affecting the expected returns of a commodity is not an easy task in practice.

Some Interesting Case Studies

SOFR Futures

The London Inter-Bank Offered Rate in US dollars, or LIBOR USD, was for half a century the reference rate for corporate loans denominated in USD. LIBOR was also quoted in other major currencies such as the Japanese Yen (JPY) or the Swiss Franc (CHF), for example. Because of how LIBOR was computed and the risks it could pose to the global financial system, shortly after the 2008 financial crisis global regulators convene to transition away from LIBOR.

In 2014, the Federal Reserve Board and the New York Fed convened the Alternative Reference Rates Committee (ARRC), a group of private-market participants tasked with identifying robust alternatives to USD LIBOR and supporting a transition away from LIBOR. In 2017, the ARRC selected the Secured Overnight Financing Rate (SOFR) as a viable replacement from LIBOR USD.

SOFR is a secured overnight interest rate based on Treasury repurchase transactions (repos). The repo market is liquid and accurately represents the risk-free cost of funding of major banks. Unlike LIBOR, which was a term rate, SOFR is by definition an overnight rate. As such, a very active SOFR futures market has emerged. SOFR futures are cash settled and their values are computed against SOFR rates. For 1-month SOFR futures the reference index is an arithmetic average whereas for 3-month SOFR futures the reference index is a geometric average of SOFR rates.

The notional value of the 3-month SOFR futures is equal to \$2,500 $\times$ the contract grade IMM index, which is computed as 100 minus the business-day compounded SOFR per annum during the contract reference quarter. For 1-month SOFR futures, the notional value is computed similarly but against an arithmetic SOFR during the contract delivery month.

Negative Oil Spot Prices

The COVID-19 pandemic hit countries hard in March 2020. As many economies came to a halt, oil inventories rose and refineries reached storage capacity. The buyer of an Apr 20 light sweet crude oil futures contract is required, if the contract reaches maturity, to purchase 1,000 barrels of oil. On April 20, 2020 there were many traders left with long positions that did not want to hold them until maturity the next day. In order to get out of a long futures position, you need to sell the same contract and that would net out the long position. The problem is that in order to sell an existing futures contract someone must be willing to buy it, and no trader wanted to purchase more contracts that day. Indeed, it would have meant to purchase physical oil for which there was no place to store it.

As a consequence, many sellers that day were willing to pay the buyer so they could get out of their long positions. In other words, the price of spot oil became negative. The figure below shows that the evolution of the spot price of oil from January 2019 until November 2021. We can see how on 4/20/2020 the spot price of oil reached almost negative $40 a barrel.

The cash price of oil, however, was not negative as no refinery would pay traders to buy the oil they had in storage.

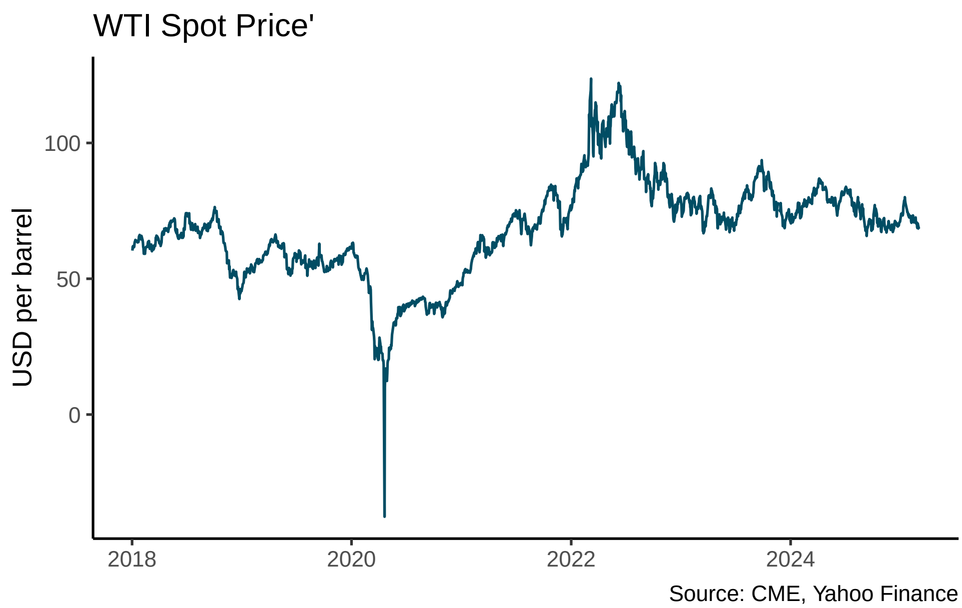

Figure 4: The figure shows the evolution of the spot price of light sweet crude oil over time.

Margin Account on the E-mini S&P 500

The E-mini S&P 500 futures contract is one of the most liquid and actively traded futures in the world. The contract value is defined as $50 × the value of the S&P 500 Index. The way the margin works on this contract is as follows.

Say you deposit $12,000 in your margin account and buy one S&P 500 E-mini futures at $4,645.00. The next day the futures price increases to $4,656.75, which is a gain of $11.75 with respect to the previous settlement. That day, your account is then credited $50 \times 1.75 =$ $587, which increases your margin to $12,587.00. If the day after the futures price decreases to \$4,652.25, then your account will lose $50 \times 4.5 = \$225.00$, bringing your margin down to \$12,362.00.

The table below describes these transactions.

Day	Futures Price	Gain/Loss	Margin Account
0	4,645.00		12,000.00
1	4,656.75	587.50	12,587.50
2	4,652.25	-225.00	12,362.50
3	4,658.50	312.50	12,675.00

Note that futures exchanges require the margin account to be at all times above a certain minimum. If the margin account goes below the minimum margin requirement the trader will receive a margin call.

Practice Problems

Solutions to all problems can be found at lorenzonaranjo.com/fin451.

Problem 1. A stock index currently stands at 350. The risk-free interest rate is 8% per year with continuous compounding and the dividend yield on the index is 4% per year. What should be the futures price for a 4-month contract?

Problem 2. Consider an E-mini S&P 500 futures contract. Remember that the contract value is defined as \$50 times the value of the S&P 500 Index. Yesterday's futures price settlement was 4,296.12, whereas today, the S&P 500 futures price settled at 4,175.20. Compute the change in a trader's margin account if she bought one futures yesterday.

Problem 3. Assume that the risk-free interest rate is 9% per annum with continuous compounding and that the dividend yield on a stock index varies throughout the year. In February, May, August, and November, dividends are paid at a rate of 5% per annum. In other months, dividends are paid at a rate of 2% per annum. Suppose that the value of the index on July 31 is 1,300. What is the futures price for a contract deliverable in December 31 of the same year?

Problem 4. The spot price of silver is $9 per ounce. The storage costs are $0.24 per ounce per year payable quarterly in advance. Assuming that interest rates are 10% per annum for all maturities, calculate the futures price of silver for delivery in 9 months.