Properties of European Options

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Building a Covered Call

- Consider European call and put options with strike K and maturity T written on a non-dividend paying stock.
- There is also a zero-coupon risk-free bond with face value K and same maturity as the options.
- Strategy A: Long stock and short call

$$\text{Cost} = S - C$$

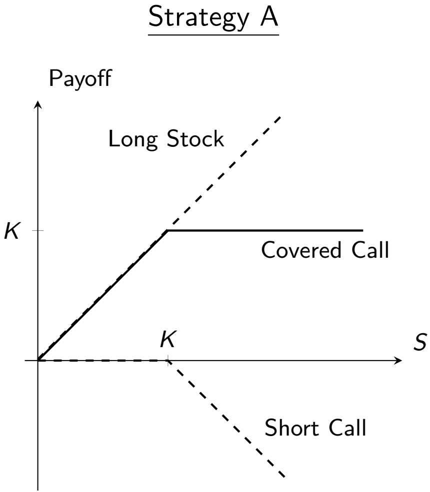
$$\text{Payoff} = \begin{cases} S_T & \text{if } S_T \leq K \\ K & \text{if } S_T > K \end{cases}$$

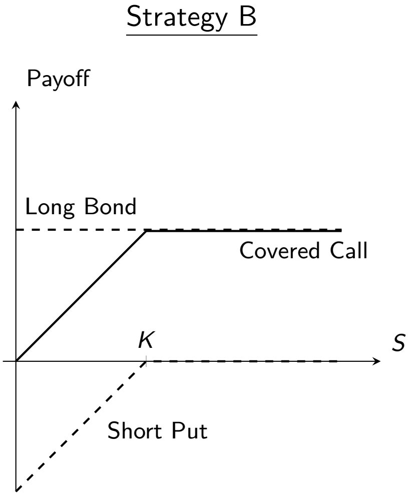
- Strategy B: Long bond and short put

$$\text{Cost} = Ke^{-rT} - P$$

$$\text{Payoff} = \begin{cases} S_T & \text{if } S_T \leq K \\ K & \text{if } S_T > K \end{cases}$$

Payoff Diagrams for Both Strategies





Put-Call Parity

- Since both strategies have the same payoff, they should have the same price.
  - Otherwise, buy the cheapest strategy and sell the most expensive one.
  - This would generate a free positive cash flow with zero risk.
- For European options written on non-dividend paying stocks, following relationship known as put-call parity must hold:

$$S - C = Ke^{-rT} - P$$

Example 1

- Consider a non-dividend paying stock trading at \$110 and assume that the continuously-compounded risk-free rate is 5% per year.
- A European call option with strike price \$110 and maturity 9 months trades for \$13.30.
- Then, according to put-call parity, we should have that a European put with the same strike and maturity as the call should cost:

$$\begin{aligned} P &= C - S + Ke^{-rT} \\ &= 13.30 - 110 + 110e^{-0.05\times 0.75} \\ &= 9.25 \end{aligned}$$

Example 2

- What if in the previous example everything stays the same, but you find that the put trades for \$9?
  - Then we have an arbitrage opportunity!
- Let us consider both strategies discussed previously that we know have the same payoff.
- Strategy A: Long stock and short call

$$\text{Cost}_A = 110 - 13.30 = 96.70$$

- Strategy B: Long bond and short put

$$\text{Cost}_B = 110e^{-0.05 \times 0.75} - 9 = 96.95$$

- Since $\text{Cost}_A < \text{Cost}_B$, we should buy A and sell B which generates an instant profit of \$0.25 per share.

Example 2 (cont'd)

Long Stock	-110.00
Short Call	13.30
Short Bond	105.95
Long Put	-9.00
Total	0.25

$S_T > 110$

Long Stock	S_T
Short Call	$-S_T + 110$
Short Bond	-110
Long Put	0
Total	0

$S_T \leq 110$

Long Stock	S_T
Short Call	0
Short Bond	-110
Long Put	$110 - S_T$
Total	0

Put-Call Parity and Protective Puts

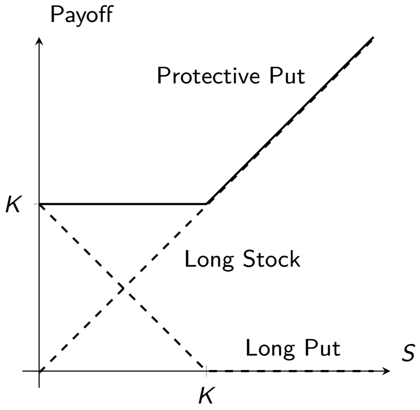
- We can also express put-call parity in the following way:

$$S + P = Ke^{-rT} + C$$

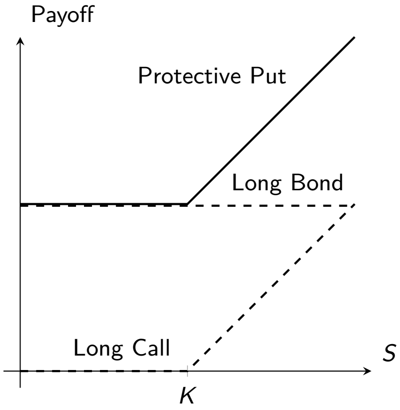
- The left hand-side of this expression is the cost of a covered put, i.e. long stock and long put.
- The right hand-side says that a protective put can also be built by buying a bond and a call.

Payoff Diagrams for Protective Put

Strategy A



Strategy B



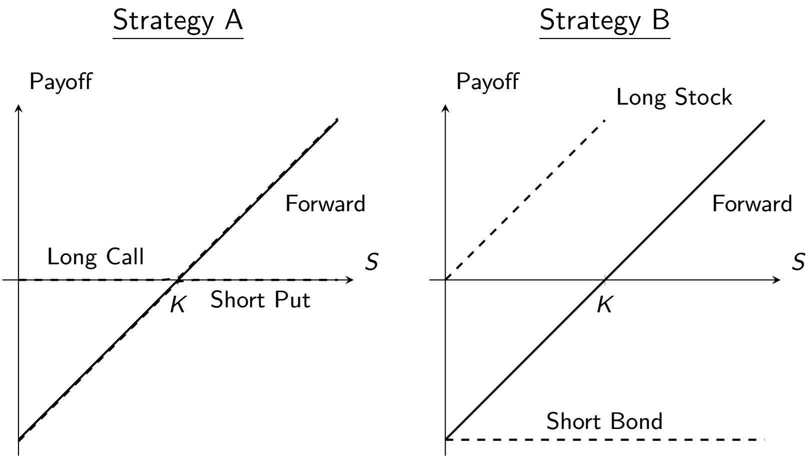
Put-Call Parity and Forward Contracts

- We can express put-call parity in yet a different way:

$$C - P = S - Ke^{-rT}$$

- The right hand-side of this expression is the cost of a forward contract with forward price K.
- The left hand-side says that a forward contract can be synthesized by buying a call and selling a put.

Payoff Diagrams for Forward Contract



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A Simple Lower Bound on Call and Put Options

- The price of a European call or put option must be positive.
- If not, any trader would like to get as many contracts as possible.
  - The worst-case scenario is that the options expire out-of-the-money in which case the payoff is zero.
  - Otherwise the options expire in-the-money and the option trader gets a positive payoff.
  - This would clearly be a nice arbitrage opportunity!
- Hence, we must have that $C \geq 0$ and $P \geq 0$.
  - Note that if $T > 0$ then it must also be the case that:
    - $C > 0$ if $S > 0$
    - $P > 0$ for any $S \geq 0$

Lower Bound on European Call Options

- Remember that we are analyzing an underlying asset that does not pay dividends.
- Put-call parity and the fact that $P \geq 0$ implies that:

$$C = P + S - Ke^{-rT} \geq S - Ke^{-rT}$$

- Given that we also have $c \geq 0$, it must the case that:

$$C \geq \max(S - Ke^{-rT}, 0)$$

Example 3

- Consider a non-dividend paying stock where $S = 110$ and $r = 5\%$ per year with continuous compounding.
- Consider a call option with strike \$110 and maturity 9 months.
- It must be the case that:

$$C \geq \max(110 - 110e^{-0.05\times 0.75}, 0) = \max(4.05, 0) = 4.05$$

- Hence, no matter how low the volatility is on this European call option, its premium must be higher than \$4.05.
  - Otherwise it might be possible to synthesize a negative price put.

Upper Bound on European Call Options

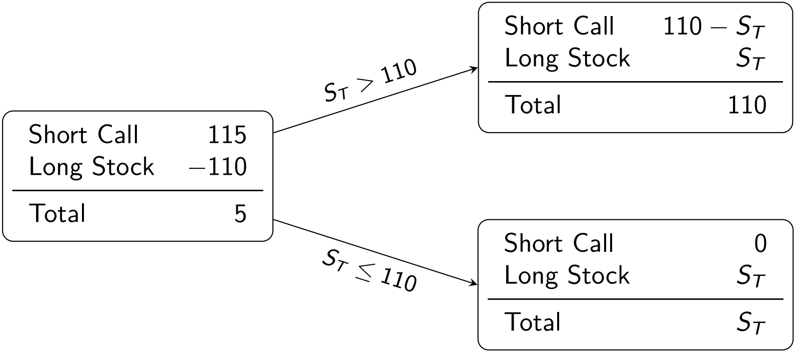
- On the other hand, the price of a European call on a non-dividend paying asset must cost less than the stock itself:

$$C \leq S$$

- If not, it would make sense to write a call and use part of the proceeds to buy a share of stock.

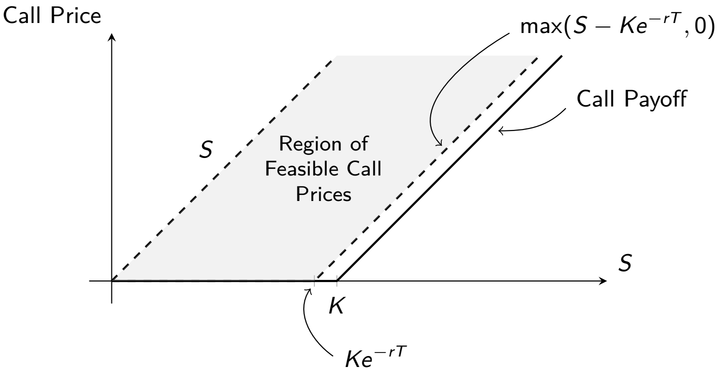
Example 4

- Assume that $S = 110$, $K = 110$, $T = 0.75$ years and $C = 115$.



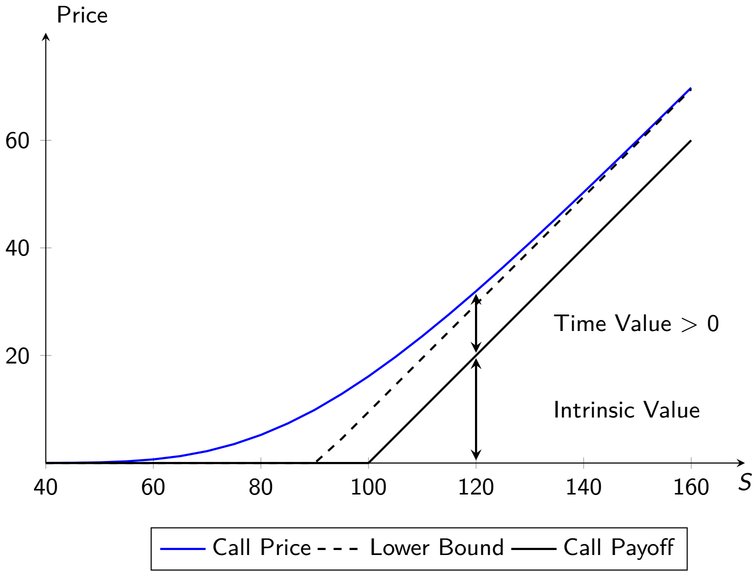
- This strategy makes money for free at $T = 0$ and keeps making money at $T = 0.75$!

Feasible Prices for European Call Options



The graph describes the region of feasible prices for European call options written on a non-dividend paying asset when the risk-free rate is *positive*.

Time Value for European Call Option ($r > 0$)



Bounds on European Put Options

- Put-call parity and the fact that $p \geq 0$ implies that:

$$P = C - S + Ke^{-rT} \geq -S + Ke^{-rT}$$

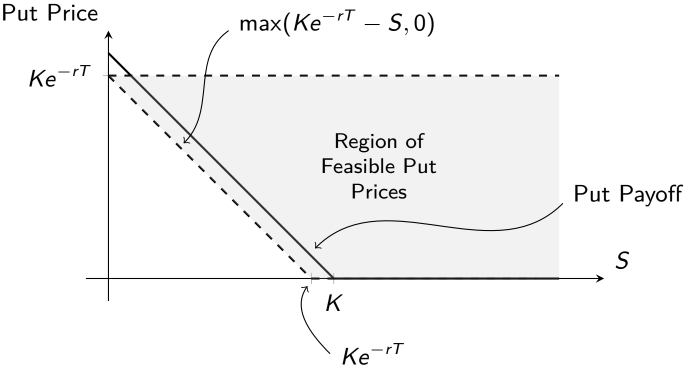
- Given that we also have $p \geq 0$, it must the case that:

$$P \geq \max(Ke^{-rT} - S, 0)$$

- Also, the maximum amount of money one can lose by writing a European put is K, which in present value terms is equal to Ke^{-rT}, implying that:

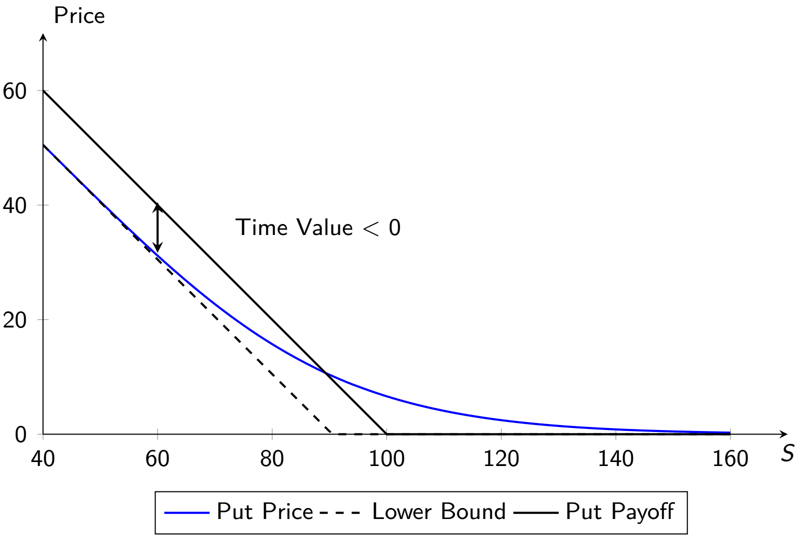
$$P \leq Ke^{-rT}$$

Feasible Prices for European Put Options



The graph describes the region of feasible prices for European put options written on a non-dividend paying asset when the risk-free rate is positive.

Time Value of European Put Option ($r > 0$)



Summary

European Options Bounds

We have the following bounds for European call and put options written on a non-dividend paying asset:

$$
\begin{aligned}
\max(0, S - Ke^{-rT}) &\leq C \leq S \\
\max(0, Ke^{-rT} - S) &\leq P \leq Ke^{-rT}
\end{aligned}
$$

Example 5

- The risk-free rate is $r = 10\%$ per year with continuous compounding.
- Furthermore, assume that $S = 50$, $K = 45$, and $T = 1.20$.
- Let us compute the bounds for European call and put options.
  - First, for the put option we have that:

$$0 = \max(45e^{-0.10\times1.20} - 50, 0) \leq P \leq 45e^{-0.10\times1.20} = 39.91$$

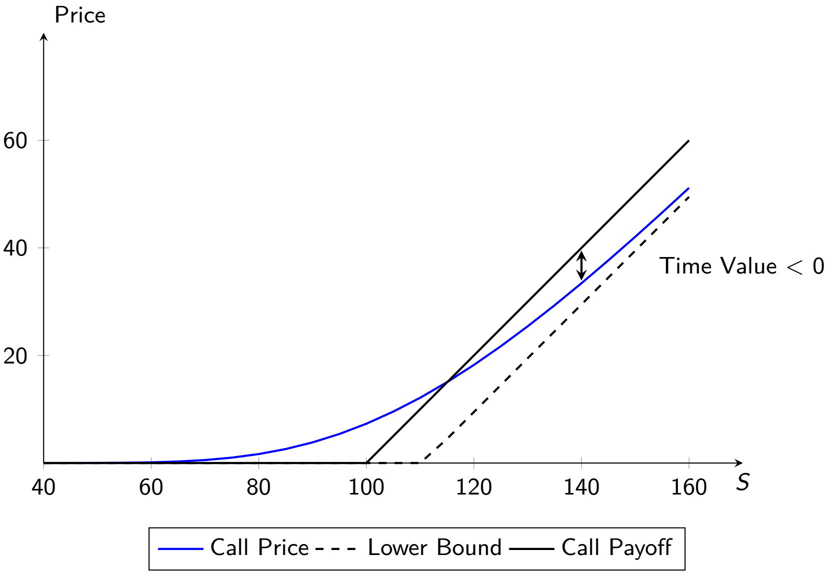
  - Second, for the call option:

$$10.09 = \max(50 - 45e^{-0.10\times1.20}, 0) \leq C \leq 50$$

Impact of Negative Interest Rates

- During the last decade, interest rates in many countries became negative, even for maturities longer than 10 years.
- For a European call option, when $r < 0$ its lower bound is less than its intrinsic value, i.e., for a sufficiently high S the option will have negative time value and it might be optimal to early exercise an American call option.
- For put options, negative interest rates means that a European put option always has positive time value, i.e., it is not optimal to exercise early an American put option.
- Standard results that are usually taught in derivative courses get reversed!

Time Value of European Call Option ($r < 0$)



Time Value of European Put Option ($r < 0$)