Exotic Options

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Outline

1. Packages
2. Variations of the Black & Scholes Framework
3. Path-Dependent Options
4. Other Exotic Options

Packages

- A package is a portfolio of standard options.
- The main difference between a package and an option strategy is that the package is sold as a whole product, whereas an option strategy involves trading different options at the same time.
- We have studied many option strategies such as bull spreads, bear spreads, straddles, strangles, butterflies, and condors, which could be sold as a package.

Range Forward Contracts

- One popular package is a range forward contract.
- Have the effect of ensuring that the exchange rate paid or received will lie within a certain range.
- When currency is to be paid, it involves selling a put with strike K_1 and buying a call with strike K_2 (with $K_1 < K_2$).
  - This would be similar to a long forward position.
- When currency is to be received it involves buying a put with strike K_1 and selling a call with strike K_2.
  - This would be similar to a short forward position.
- Normally the price of the put equals the price of the call so the contract has zero cost.

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Gap Options

- A gap call pays $S_T - K_1$ when $S_T > K_2$, and zero otherwise.
- A gap put pays $K_1 - S_T$ when $S_T < K_2$, and zero otherwise.
- We can adapt our previous analysis to get:

$$\text{Gap Call} = Se^{-qT}\,\Phi(d_1) - K_1 e^{-rT}\,\Phi(d_2)$$
$$\text{Gap Put} = K_1 e^{-rT}\,\Phi(-d_2) - Se^{-qT}\,\Phi(-d_1)$$

where

$$d_1 = \frac{\ln(S/K_2) + (r - q + 0.5\sigma^2)T}{\sigma\sqrt{T}}$$
$$d_2 = d_1 - \sigma\sqrt{T}$$

Binary Options

- A cash-or-nothing call pays Q if $S_T > K$, otherwise pays nothing.

$$\text{Value} = Qe^{-rT}\,\Phi(d_2)$$

- A cash-or-nothing put pays Q if $S_T < K$, otherwise pays nothing.

$$\text{Value} = Qe^{-rT}\,\Phi(-d_2)$$

- An asset-or-nothing call pays S_T if $S_T > K$, otherwise pays nothing.

$$\text{Value} = Se^{-qT}\,\Phi(d_1)$$

- An asset-or-nothing put pays S_T if $S_T < K$, otherwise pays nothing.

$$\text{Value} = Se^{-qT}\,\Phi(-d_1)$$

Forward Start Options

- The option starts at a future time τ and expires at time $T > \tau$.
- Implicit in employee stock option plans.
- Often structured so that strike price equals asset price at time τ, that is, $K = S_\tau$.
- Therefore, the value of a call or put option at time τ is:

$$C_\tau = S_\tau e^{-q(T-\tau)}\,\Phi(d_1) - S_\tau e^{-r(T-\tau)}\,\Phi(d_2)$$
$$P_\tau = S_\tau e^{-r(T-\tau)}\,\Phi(-d_2) - S_\tau e^{-q(T-\tau)}\,\Phi(-d_1)$$

where $d_1 = \dfrac{(r - q + 0.5\sigma^2)(T-\tau)}{\sigma\sqrt{T-\tau}}$ and $d_2 = d_1 - \sigma\sqrt{T-\tau}$.

Valuing a Forward Start Option

- The price of the option today is just $V_0 = \mathrm{E}(V_\tau)e^{-r\tau}$ where the expectation is taken of course with respect the risk-neutral measure.
- Noting that the futures price of a contract expiring at time τ is given by $f = \mathrm{E}(S_\tau) = Se^{(r-q)\tau}$, we have that $\mathrm{E}(S_\tau)e^{-r\tau} = Se^{-q\tau}$.
- Therefore:

$$C = \left(Se^{-q(T-\tau)}\,\Phi(d_1) - Se^{-r(T-\tau)}\,\Phi(d_2)\right)e^{-q\tau}$$
$$P = \left(Se^{-r(T-\tau)}\,\Phi(-d_2) - Se^{-q(T-\tau)}\,\Phi(-d_1)\right)e^{-q\tau}$$

where $d_1 = \dfrac{(r-q+0.5\sigma^2)(T-\tau)}{\sigma\sqrt{T-\tau}}$ and $d_2 = d_1 - \sigma\sqrt{T-\tau}$.

- We can then see that the value of a forward start option is $e^{-q\tau}$ times the value of similar option starting today.

Cliquet Option

- A series of call or put options with rules determining how the strike price is determined.
- For example, a cliquet might consist of 20 at-the-money three-month options. The total life would then be five years.
- When one option expires a new similar at-the-money is coming into existence.
- As you can see, this would be a portfolio of 20 forward starting options that we just saw how to value.

Chooser Options

- Option starts at time 0 and matures at T.
- At time τ $(0 < \tau < T)$ the buyer chooses whether it is a put or call with strike K and expiring at T, at which point the value of the chooser is $\max(C_\tau, P_\tau)$.
- From put-call parity:

$$P_\tau = C_\tau + Ke^{-r(T-\tau)} - S_\tau e^{-q(T-\tau)}$$

which implies that:

$$\max(C_\tau, P_\tau) = C_\tau + e^{-q(T-\tau)} \max\left(Ke^{-(r-q)(T-\tau)} - S_\tau, 0\right)$$

- This is the payoff of a call with strike K and expiring at T plus $e^{-q(T-\tau)}$ puts with strike $\widetilde{K} = Ke^{-(r-q)(T-\tau)}$ and expiring at time τ.

Compound Option

- Option to buy or sell an option.
- We have therefore four possible combinations:
  - Call on call
  - Put on call
  - Call on put
  - Put on put
- These options can be valued analytically (we will not cover this in class, though).
- Intuitively, the price of such options is quite low compared with the underlying option.

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Lookback Options

- A floating lookback call pays $S_T - S_{min}$ at time T.
  - Allows the buyer to buy the stock at the lowest observed price in some interval of time.
- A floating lookback put pays $S_{max} - S_T$ at time T.
  - Allows the buyer to sell the stock at the highest observed price in some interval of time.
- A fixed lookback call pays $\max(S_{max} - K, 0)$ at time T.
  - Like a regular call but the final payoff depends on the maximum value of the stock during the lifetime of the option.
- A fixed lookback put pays $\max(K - S_{min}, 0)$ at time T.
  - Like a regular put bu the final payoff depends on the minimum value during the life of the option.
- It is possible to derive analytic formulas for all types.

Asian Options

- The payoff of such options is related to the average stock price $\bar{S}$ from time 0 until T.
- Average price options pay:
  - Call: $\max(\bar{S} - K, 0)$
  - Put: $\max(K - \bar{S}, 0)$
- Average strike options pay:
  - Call: $\max(S_T - \bar{S}, 0)$
  - Put: $\max(\bar{S} - S_T, 0)$
- No exact analytic valuation, but can be approximately valued by assuming that the average stock price is lognormally distributed.

Barrier Options

- Barrier options are either call or put options that get activated or deactivated depending on whether the stock hits a barrier from above or below.
    - "In" options come into existence only if stock price hits the barrier before option maturity.
    - "Out" options die if stock price hits the barrier before option maturity.
    - "Up" options require that the stock hits the barrier from below.
    - "Down" options require that the stock hit the barrier form above.
- Therefore, there are eight possible combinations.

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Exchange Options

- Option to exchange one asset for another.
- For example, an option to exchange one unit of U for one unit of V.
- Payoff is then $\max(V_T - U_T, 0)$.

Basket Options

- A basket option is an option to buy or sell a portfolio of assets.
- This can be valued by calculating the first two moments of the value of the basket at option maturity and then assuming it is lognormal.

Non-Standard American Options

- Exercisable only on specific dates (Bermudans)
- Early exercise allowed during only part of life (initial “lock out” period)
- Strike price changes over the life (warrants, convertibles)