

Options on Futures

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Options on Futures

- The underlying asset is a futures contract, usually on a commodity, a precious metal, a currency, an interest rate or a bond.
- These options usually expire on or a few days before the earliest delivery date of the underlying futures contract.
- If a call futures option is exercised, the buyer gets a long position in the futures plus a cash amount equal to the excess of the futures price at the time of the most recent settlement over the strike price.
- If a put futures option is exercised, the buyer acquires a short position in the futures plus a cash amount equal to the excess of the strike price over the futures price at the time of the most recent settlement.

Example 1

A September call option contract on copper futures has a strike of 425 cents per pound. A CME contract is written on 25,000 pounds of copper. The call is exercised when the futures price is 426.95 cents and the most recent settlement is 426.45 cents. The trader receives a long September futures contract on copper and $25,000 \times (4.2645 - 4.2500) = \362.50 .

Example 2

A September put option contract on soybean futures has a strike price of 1380 cents per bushel. A CME contract is written over 5,000 bushels. The put is exercised when the futures price is 1372 cents per bushel and the most recent settlement price is 1365 cents per bushel. The trader receives a short September futures contract on soybean and $5,000 \times (13.80 - 13.65) = \750 in cash.

Futures vs. Spot Options

- Futures contracts are easier to trade and more liquid than the underlying asset, as would be the case for commodities, for example.
- The exercise of the option does not lead to delivery of underlying asset, but of the futures contract on the asset which can be bought or sold easily.
- Futures options and futures usually trade on same exchange, i.e., Chicago Mercantile Exchange (CME).
- By trading on exchanges, futures options are more liquid and may entail lower transactions costs.

The Risk-Neutral Process for the Futures Price

- The futures price is defined as the expected spot price under the risk-neutral measure:

$$f_t = E_t(S_T) = S_t e^{(r-q)(T-t)}$$

- We note that

$$\begin{aligned}df &= \frac{\partial f}{\partial S} dS + \frac{1}{2} \frac{\partial^2 f}{\partial S^2} (dS)^2 + \frac{\partial f}{\partial t} dt \\&= (r - q) f dt + \sigma f dW - (r - q) f dt \\&= \sigma f dW\end{aligned}$$

- In other words, the futures price is a martingale under the risk-neutral measure.

Black's Model for European Futures Options

- We note that:

$$df = (r - r)fdt + \sigma fdW$$

- This means that a European futures option can be priced “as if” the futures pays a dividend yield equal to r :

$$C = fe^{-rT} \Phi(d_1) - Ke^{-rT} \Phi(d_2)$$

$$P = Ke^{-rT} \Phi(-d_2) - fe^{-rT} \Phi(-d_1)$$

where f is the current futures price, and:

$$d_1 = \frac{\ln(f/K) + \frac{1}{2}\sigma^2 T}{\sigma\sqrt{T}} \text{ and } d_2 = d_1 - \sigma\sqrt{T}.$$

- Therefore, European futures options and spot options are equivalent when futures contract matures at the same time as the option.

Black's Model in Practice

- Black's model is frequently used to value European options on the spot price of an asset in the over-the-counter market (OTC).
- This avoids the need to estimate income on the asset since we can use the futures price directly in the formula.

Example 3

Consider a 6-month European call option on spot gold. The 6-month futures price is \$1,806, the 6-month risk-free rate is 1% per year continuously-compounded, the strike price is \$1,820, and the volatility of the futures price is 20% per year. The option is priced using Black's model with $f_0 = 1806$, $K = 1820$, $r = 0.01$, and $\sigma = 0.20$:

$$d_1 = \frac{\ln(1806/1820) + \frac{1}{2}(0.20)^2(0.5)}{0.20\sqrt{0.5}} = 0.0161 \Rightarrow \Phi(d_1) = 0.5064$$

$$d_2 = 0.0161 - 0.20\sqrt{0.5} = -0.1253 \Rightarrow \Phi(d_2) = 0.4501$$

$$C = 1806e^{-0.01(0.5)}(0.5064) - 1820e^{-0.01(0.5)}(0.4501) = 94.88$$

The value of the call is \$94.88.

Futures Style Options

- A futures-style option is a futures contract on a futures option.
- One of the advantages of these contracts is that the margining of the instrument is that of a futures contract, i.e., futures-style margining.
- If we denote by c the futures price for a call futures-style option and by C the price of the underlying asset, which in this case is a call futures option, we have that:

$$c = Ce^{rT} = f \Phi(d_1) - K \Phi(d_2)$$

- Similarly, the futures price of a put futures-style option is:

$$p = Pe^{rT} = K \Phi(-d_2) - f \Phi(-d_1)$$