Derivatives Contracts

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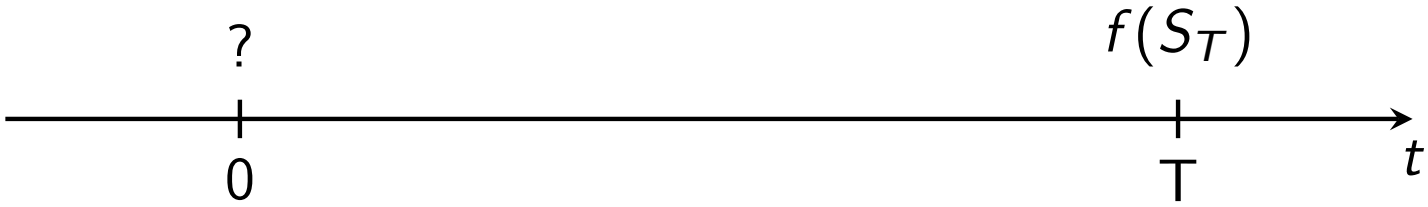
Definitions

- In this class we study the pricing, hedging and uses of **financial derivatives** or derivatives for short.
- A **derivative** is a financial instrument whose payoff depends on, or is derived from, the value of another financial asset such as a stock, a foreign currency, a futures, or another quantity such as volatility.
- The value of a derivative is then the discounted value of its payoff.
  - Linear payoffs are easier to price.
  - Non-linear payoffs are harder to value.
- A positive payoff means that you receive money, whereas a negative payoff represents an outflow of money.

A More Formal Definition

- For many derivatives, the payoff is realized at maturity.
  - Time 0 is where we are right now.
  - Time T is when the derivative expires.
- If S_T denotes the value of a stock at maturity, the payoff of the derivative is a function of S_T denoted as $f(S_T)$.
- An important question that we answer in this class is how to **price** this derivative.



Example: Derivative with Linear Payoff

- A **forward** contract is a commitment to purchase or sell an asset at maturity for a certain price K.
- The payoff of a **long forward** is the difference between the price of the asset at maturity and the price agreed in the contract, that is, the payoff is a linear function of the stock price:

$$f(S) = S - K$$

- Because the payoff can be positive or negative depending of the sign of S, it is possible for the value of the contract to be positive or negative,
  - Usually the contract is designed so the value at inception is zero.
  - Later on, the value of the contract will change and might become positive or negative.

Example: Derivative with Nonlinear Payoff

- An **option** gives the holder the right but not the obligation to purchase or sell an asset at maturity for a given price K.
- The payoff of an option is a nonlinear function of the asset price at maturity.
- For example, the buyer of a call option receives:

$$f(S) = \begin{cases} 0 & \text{if } S < K \\ S - K & \text{if } S \geq K \end{cases}$$

- Since the payoff is non-negative, the holder of an option must pay a premium to the seller.

Derivatives with Periodic Payments

- Some derivatives involve the payment of cash flows periodically over time.
- For example, **interest rates swaps** involve the exchange of a fixed interest rate for a floating interest rate, or vice-versa.
- Another example is **credit default swaps** (CDS) which involve the exchange of periodic payments in exchange for protection in case of a bond default.

Assets with Embeded Derivatives

- It is also possible to embed derivatives to simpler assets such as bonds.
- For example, many bonds found in financial markets are **callable**, that is, the issuer has the right to pay the bond holder the principal at any time before maturity.
- Other bonds are convertible into shares of the issuing company at a fixed price.
- Thus, **convertible** bonds contain a call option on the company stock which might be very valuable.

Do We Need Other Payoffs?

- In theory, we could design a derivative with any payoff function $f(S)$.
- For example, we could choose $f(S) = S^2$ or $f(S) = \ln(S)$.
- It turns out that with forwards and options it is possible to build any type of payoff that a trader might want.
- We will see that by having options and forwards with different strikes we can **complete the market**.
- Combining options and forwards together is usually called **options strategies**.

Purposes of Derivatives

- Derivatives allow investors to obtain payoffs that might be useful to achieve certain objectives.
- For example, some commodity producers use derivatives to **hedge** their future production by fixing today the price at which they will sell in the future.
- Other traders like derivatives because they can obtain custom design payoffs that allow them to **speculate** in very specific ways.
- Therefore, derivatives make both types of traders, hedgers and speculators, better off by expanding their trading opportunity set and thus increasing their utility.

The Market for Derivatives

- The demand side is composed of buy-side traders that want to use derivatives for either hedging or speculative purposes.
- The supply side is determined by sell-side traders or **market makers** that provide liquidity to the rest of the market.
- The **net demand**, which can be positive or negative, is balanced by market makers.
- In order for market makers to **hedge** their exposure, they need to dynamically trade the underlying asset and risk-free bonds.

Pricing and Hedging of Derivatives

- One of the main results in modern asset pricing is that a perfectly hedged portfolio should earn the risk-free rate of interest.
- Otherwise there would be an **arbitrage opportunity**.
- Therefore, in order to price an option or a forward contract we need to learn how to hedge or replicate the position first.
- For options, the hedging recipe depends heavily on the modelling of the stock price evolution over time.
  - Time can be seen as either discrete or continuous.
  - The distribution of random shocks will affect the evolution of stock prices over time.