Properties of European Options
The Impact of Dividends

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Put-Call Parity with Dividends

- For European options written on dividend paying stocks, the put-call parity is modified as follows:

$$C - P = S - D - Ke^{-rT}$$

  where D is the present value of dividends paid during the life of the option.
- In order to derive this expression we will proceed as before by trying to build a covered call in two different ways.

Building a Covered Call

- Supose that we did the same as in a previous lecture.
- Strategy A: Long stock and short call

$$\text{Cost} = S - C$$

$$\text{Payoff} = \begin{cases} S_T + FV(D) & \text{if } S_T \leq K \\ K + FV(D) & \text{if } S_T > K \end{cases}$$

- Strategy B: Long bond and short put

$$\text{Cost} = Ke^{-rT} - P$$

$$\text{Payoff} = \begin{cases} S_T & \text{if } S_T \leq K \\ K & \text{if } S_T > K \end{cases}$$

- Both strategies no longer have the same payoff at maturity because the stock pays dividends.

Adjusting Strategy A

- Strategy A: Long stock, borrow D and short call

$$\text{Cost} = S - D - C$$

$$\text{Payoff} = \begin{cases} S_T & \text{if } S_T \leq K \\ K & \text{if } S_T > K \end{cases}$$

- Strategy B: Long bond and short put

$$\text{Cost} = Ke^{-rT} - P$$

$$\text{Payoff} = \begin{cases} S_T & \text{if } S_T \leq K \\ K & \text{if } S_T > K \end{cases}$$

- Note that in A we use the dividends, reinvested at r, to repay the loan and generate the same payoff as in B.

Example 1

- Suppose that $S = 110$, $r = 5\%$, $K = 110$, $T = 9$ months, and $C = 13.30$.
- The stock is expected to pay dividends of \$2 in 6 months, and \$2.5 in 1 year.
- What should be the no-arbitrage price of a European put with the same strike and maturity as the European call?
- The present value of the relevant dividends is:

$$D = 2e^{-0.05\times 6/12} = 1.95$$

- Then, according to put-call parity we should have that:

$$P = 13.30 - 110 + 1.95 + 110e^{-0.05\times 9/12} = 11.20$$

Example 2

- What if in the previous example everything stays the same, but you find that the put trades for $11?
  - Then we have an arbitrage opportunity since the put is relatively cheap compared to what it should trade.
  - Hence, we should buy the put and sell the synthetic put. Note that the stock will pay a dividend of $2 in six months that can be used to pay the loan at that time.

	$T = 0$	$T = 6/12$	$T = 9/12$	
			$S_T \leq 110$	$S_T > 110$
Long put	-11.00	0	$110 - S_T$	0
Short call	13.30	0	0	$-(S_T - 110)$
Long stock	-110.00	2	S_T	S_T
Borrow D	1.95	-2	0	0
Short bond	105.95	0	-110	-110
Total	0.20	0	0	0

Lower Bound on European Options with Cash Dividends

- With dividends, we modify the lower bounds on European call and put options as follows:

$$C \geq \max(S - D - Ke^{-rT}, 0)$$
$$P \geq \max(Ke^{-rT} - S + D, 0)$$

- As for the case with no dividends, these results are a consequence of put-call parity and the fact the option premium is never negative.

Example 3

- Suppose that you have $S = 110$, $r = 5\%$, $K = 110$, and $T = 9$ months.
- The stock is expected to pay dividends of \$2 and \$2.5, in six and twelve months, respectively.
- The previous result implies that:

$$C \geq \max(110 - 2e^{-0.05\times 6/12} - 110e^{-0.05\times 0.75}, 0) = 2.10$$

- Note that we only include the dividend paid in 6 months since the maturity of the option is 9 months.
- Also note that the bound is lower than the bound for an otherwise equivalent option written on a non-dividend paying stock.
    - Since the call is European type, you can only purchase the stock at maturity.
    - This means that you will not receive the dividend paid in 6 months.

Upper Bound on European Options with Cash Dividends

- Since the payoffs of strategies A and B described above are positive, the cost of both strategies must be positive.
- Therefore, if the asset pays cash dividends, the upper bounds on European call and put options are as follows:

$$C \leq S - D$$
$$P \leq Ke^{-rT}$$

Example 4

- Suppose that you have $S = 110$, $r = 5\%$, $K = 110$, and $T = 9$ months.
- The stock is expected to pay dividends of \$2 and \$2.5, in six and twelve months, respectively.
- The previous result implies that:

$$C \leq 110 - 2e^{-0.05 \times 6/12} = 108.05$$

- Hence, no matter how high the volatility is on this European call option with strike \$110 and maturity 9 months, its premium must be less than \$108.05.

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The Dividend Yield

- There are many assets that pay dividends continuously, like a currency, or that can be modeled as such, like a stock index.
- In these cases it is convenient to model dividends as a percentage yield paid over time.
- We will denote the continuously-compounded dividend yield by q.
- The asset S then pays every instant t a dividend of $qS_t\Delta t$.
- Therefore, the dividend yield can be seen as the units of the asset growing over time at the rate q.
- In practice, this is the approach used to model options on stock indices and currencies, although some practitioners also use it to model individual stocks as well.

Put-Call Parity with a Dividend Yield

- For European options written on assets paying a dividend yield, the put-call parity is modified as follows:

$$C - P = Se^{-qT} - Ke^{-rT}$$

  where q is the dividend yield paid continuously by the asset during the life of the option.
- In order to derive this expression we will proceed as before by trying to build a covered call in two different ways.

Two Strategies with the Same Payoff

- Strategy A: Long e^{-qT} units of the asset and short call

$$\text{Cost} = Se^{-qT} - C$$

$$\text{Payoff} = \begin{cases} S_T & \text{if } S_T \leq K \\ K & \text{if } S_T > K \end{cases}$$

- Strategy B: Long bond and short put

$$\text{Cost} = Ke^{-rT} - P$$

$$\text{Payoff} = \begin{cases} S_T & \text{if } S_T \leq K \\ K & \text{if } S_T > K \end{cases}$$

- Note that in A the asset grows at the rate q, so the total number of "units" of the asset at maturity is $e^{-qT}e^{qT} = 1$.

Example 5

- Suppose that $S = 110$, $r = 5\%$, $q = 3\%$, $K = 110$, $T = 9$ months, and $C = 13.30$.
- What should be the no-arbitrage price of a European put with the same strike and maturity as the European call?
- According to put-call parity we should have that a European put with the same strike and maturity as the call should cost:

$$P = 13.30 - 110e^{-0.03\times 9/12} + 110e^{-0.05\times 9/12} = 11.70.$$

Example 6

- What if in the previous example everything stays the same, but you find that the put trades for $11?
- Then we have an arbitrage opportunity since the put is relatively cheap compared to what it should trade.
- Hence, we should buy the put and sell the synthetic put. Note that because the asset pays a dividend yield we need to purchase slightly less of it today in order to get one unit if the asset in nine months.

	$T = 0$	$T = 9/12$	
		$S_T \leq 110$	$S_T > 110$
Long put	−11.00	$110 - S_T$	0
Short call	13.30	0	$-(S_T - 110)$
Long e^{-qT} units of asset	−107.55	S_T	S_T
Short bond	105.95	−110	−110
Total	0.70	0	0

Lower Bound on European Options with Dividend Yields

- If the asset pays a dividend yield, we modify the lower bounds on European call and put options as follows:

$$C \geq \max(Se^{-qT} - Ke^{-rT}, 0)$$
$$P \geq \max(Ke^{-rT} - Se^{-qT}, 0)$$

- As for the case with no dividends, these results are a consequence of put-call parity and the fact the option premium is never negative.

Example 7

- Suppose that you have $S = 110$, $r = 5\%$, $q = 3\%$, $K = 110$, and $T = 9$ months.
- The previous result implies that:

$$C \geq \max(110e^{-0.03\times 9/12} - 110e^{-0.05\times 9/12}, 0) = 1.60$$

- Hence, no matter how low the volatility is on this European call option with strike \$110 and maturity 9 months, its premium must be higher than \$1.60.

Upper Bound on European Options with Dividend Yields

- Since the payoffs of strategies A and B described above are positive, the cost of both strategies must be positive.
- Therefore, if the asset pays a dividend yield, the upper bounds on European call and put options are as follows:

$$C \leq Se^{-qT}$$
$$P \leq Ke^{-rT}$$

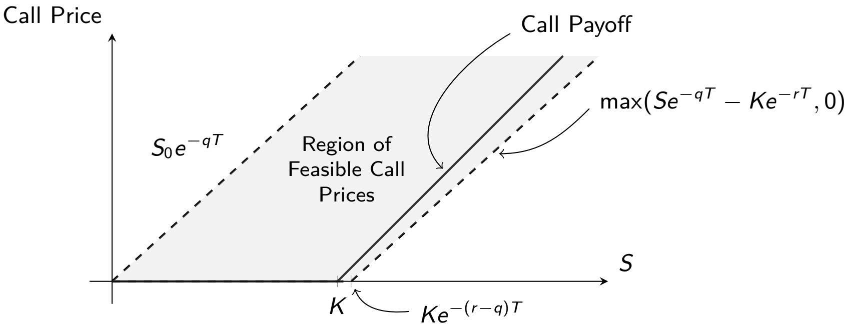
Example 8

- Suppose that you have $S = 110$, $r = 5\%$, $q = 3\%$, $K = 110$, and $T = 9$ months.
- The previous result implies that:

$$C \leq 110e^{-0.03 \times 9/12} = 107.55$$

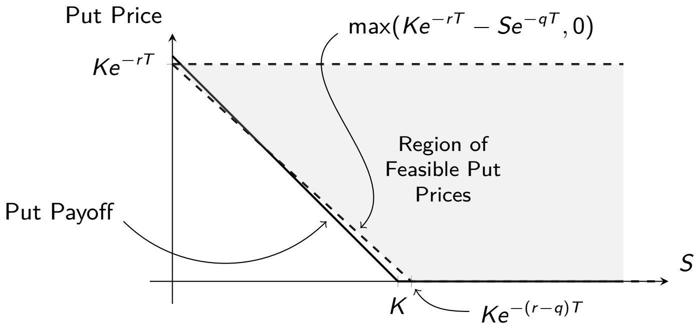
- Hence, no matter how high the volatility is on this European call option with strike \$110 and maturity 9 months, its premium must be less than \$107.55.

Feasible Prices for European Call Options



The graph describes the region of feasible prices for European call options written on an asset that pays a positive dividend yield such that $q > r$.

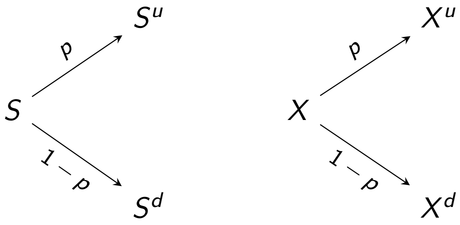
Feasible Prices for European Put Options



The graph describes the region of feasible prices for European put options written on an asset that pays a positive dividend yield such that $q > r$.

Binomial Pricing

- Pricing options when the asset pays a dividend yield requires to adjust the risk-neutral probabilities accordingly.
- Say that over the next period Δt the asset price can go up to $S^u = Su$, or down to $S^d = Sd$, and that we want to price a derivative X that pays either X^u or X^d in each state, respectively.



Risk-Neutral Pricing

- The risk-neutral probability of an up-move in this case is given by:

$$p = \frac{e^{(r-q)\Delta t} - d}{u - d}$$

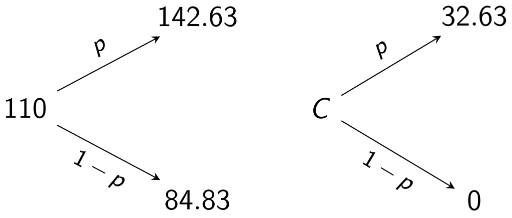
- The price of the derivative is then:

$$X = (pX^u + (1-p)X^d)e^{-r\Delta t}$$

- Note that we can make this model consistent with the Black-Scholes model by choosing $u = e^{\sigma\sqrt{\Delta t}}$ and $d = 1/u$, where σ represents the annualized volatility of the asset returns.

Example 9

- Suppose that $S = 110$, $r = 5\%$, $q = 3\%$, $\sigma = 30\%$, $K = 110$, $T = 9$ months.
- Using a one-period binomial tree, let's compute the no-arbitrage price of a European call option.
- The binomial trees for the asset and the call are as follows:



$$p = \frac{110e^{(0.05-0.03)\times 9/12} - 84.83}{142.63 - 84.83} = 0.4642$$
$$C = (32.63p + 0(1-p))e^{-0.05\times 9/12} = 14.59$$