Interest Rates

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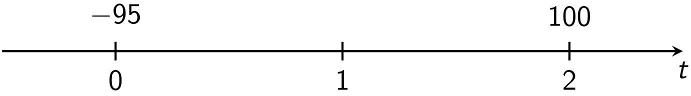
Interest Rates and Derivatives

- The pricing of financial assets is always relative to some benchmarks that we believe are well priced.
- In order to compare cash flows occurring at different points in time, we use risk-free bonds as the relevant benchmarks.
- Unfortunately, real financial markets are rarely fully integrated, so the landscape of interest rates is varied and complex.
- Moreover, when pricing options and other derivatives, it is common both by academics and practitioners to use continuous compounding to discount riskless cash flows.

Example 1

- Consider a **zero-coupon** bond that pays for certain \$100 in 2 years.
- The bond today costs \$95.



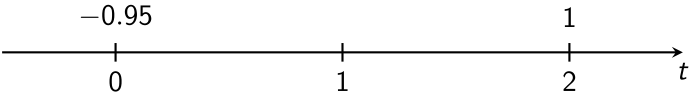
- The fact that \$100 paid in two years costs today \$95 can also be expressed as a percentage rate of return per year.

$$95(1+r)^2 = 100 \Rightarrow r = \left(\frac{100}{95}\right)^{1/2} - 1 = 2.60\%.$$

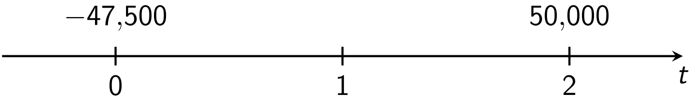
- In other words, the implicit *interest rate* paid by the 2-year zero coupon bond is 2.60% per year compounded annually.

Example 1 (continued)

- This means that every dollar paid for certain in two years today costs \$0.95.



- Thus, a bond that pays for certain \$50,000 in two years today should cost $0.95 \times 50{,}000 = \$47{,}500$.



An Arbitrage Example

- What if this new bond trades for a different price, say \$47,000?
- Then there would be an *arbitrage opportunity*.

500 bonds paying \$100 in two years

=

1 bond paying \$50,000 in two years

- How to profit?
  - Buy the bond with face value \$50,000 expiring in two years.
  - Sell 500 bonds with face value \$100 expiring at the same date.
- This long-short strategy provides today with a positive cash-flow of \$500 and is fully hedged in two years.

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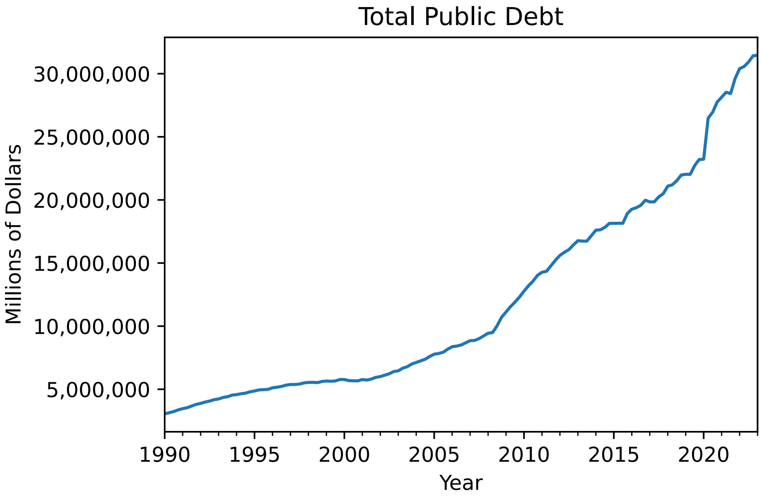
Treasury Rates

- The United States government through the Department of the Treasury issues Bills, Notes and Bonds to finance government activities.
- Treasury bonds are usually perceived as risk-free, i.e. no risk of default.
- Despite the size of the U.S. Treasury market, the demand for Treasury securities often surpasses its supply.
- For this reason, the yield-to-maturity (YTM) of Treasury bonds might be lower than the rate of a fully collateralized loan.
- Therefore, Treasury rates are commonly not used as benchmark rates to price derivative securities.

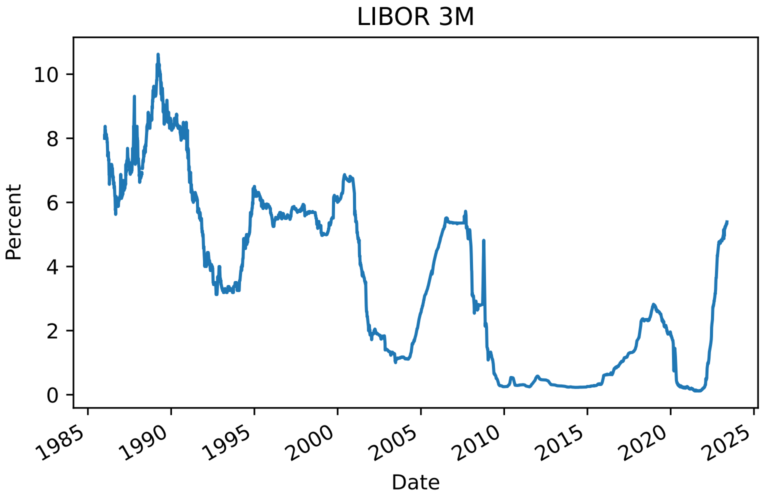
U.S. Public Debt



LIBOR

- The London Interbank Offered Rate (LIBOR) has been at the heart of the financial system for many decades.
- For all this time, LIBOR has provided a reference for pricing derivatives, loans and securities.
- Large corporate loans used to be indexed to LIBOR.
- Since many borrowers liked to pay a fixed rate, one of the most important derivatives that used LIBOR as a reference rate were interest rate swaps.
- LIBOR is no longer used and in the U.S. it has been replaced with SOFR.

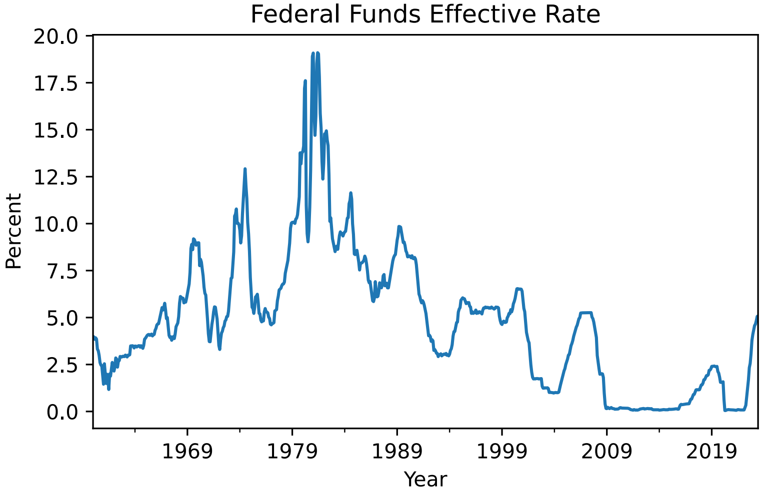
3-Month LIBOR



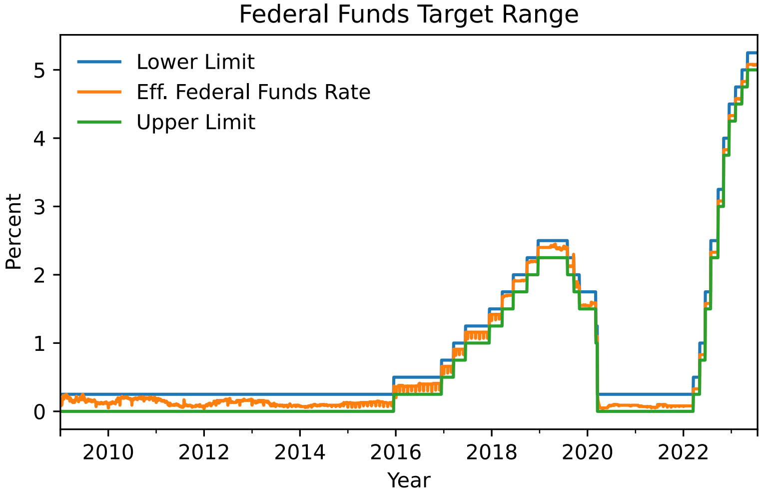
OIS and Overnight Rates

- In the United States, banks are required to maintain reserves in cash with the Federal Reserve.
- When a bank needs to increase their reserves, they usually borrow overnight from another bank that might have a reserve surplus.
- The weighted-average rate of these brokered transactions is termed the *effective federal funds rate* (EFFR).
- When the Federal Reserve determines the *target federal funds rate*, they implement their policy by making sure that the EFFR is close every day to their target.
- An *overnight indexed swap* (OIS) is an over-the-counter financial contract in which one party pays the compounded EFFR over a certain period, say three months, in exchange for a fixed payment.

Evolution of the Effective Federal Funds Rate



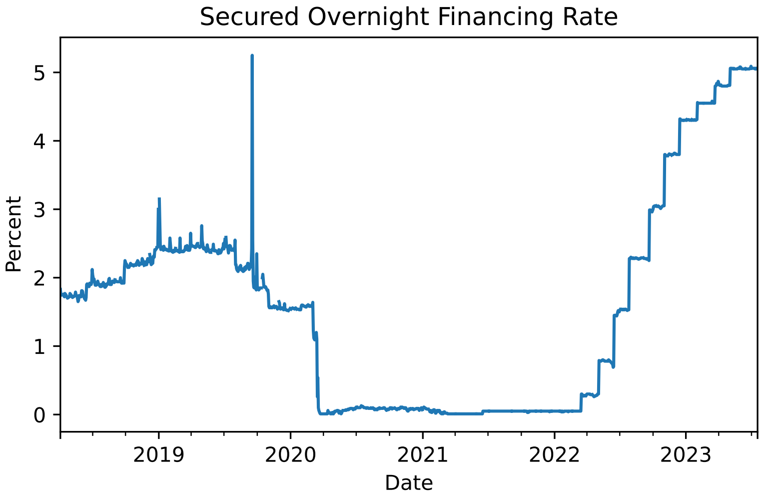
Federal Reserve Interest Rate Policy



SOFR and Repo Rates

- A repurchase agreement or repo, a financial institution or trader sells some securities to a counterparty with the agreement to repurchase them back later for a slightly higher price.
- The implicit interest rate in this transaction is the repo rate.
- Unlike LIBOR and the EFFR, repo rates are secured borrowing rates.
- The weighted average of these repo transactions is called the *secured overnight financing rate* (SOFR).
- Effective 2022, this rate has replaced LIBOR USD.

Federal Reserve Interest Rate Policy



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Compounding Multiple Times per Year

- If the interest rate is 10% per year compounded annually, after a year, \$100 invested at this rate will grow to

$$\text{FV} = 100(1.10) = \$110.00.$$

- If the same interest rate is compounded semi-annually, \$100 will grow to

$$\text{FV} = 100(1.05)^2 = \$110.25.$$

- If the same interest rate is compounded monthly, \$100 will grow to

$$\text{FV} = 100\left(1 + \frac{0.10}{12}\right)^{12} = \$110.47.$$

- If the same interest rate is compounded daily, \$100 will grow to

$$\text{FV} = 100\left(1 + \frac{0.10}{365}\right)^{365} = \$110.52.$$

Compounding in the Limit

- It turns out that there is a limit to the compounding operation we just did:

$$\lim_{n\to\infty} 100\left(1+\frac{0.10}{n}\right)^n = 100e^{0.10} = \$110.52.$$

- We call this operation *continuous* compounding, and you can see that compounding daily is already a pretty good approximation of it.
- In general, if we denote by r the continuously-compounded interest rate, the relationship between present value (PV) and future value (FV) is given by:

$$\text{FV} = \text{PV}e^{rT} \iff \text{PV} = \text{FV}e^{-rT}$$

Effective Annual Rate (EAR)

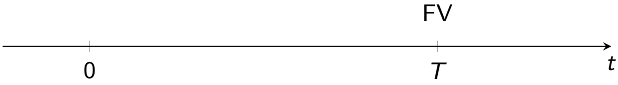
- Say that you invest \$100 in a deposit account and after a year your balance is \$110.
- The EAR of your investment in this case is 10%.
- We can also compute the equivalent continuously-compounded rate which in this case is:

$$110 = 100e^r \Rightarrow r = \ln\left(\frac{110}{100}\right) = 9.53\%.$$

- Therefore 10% per year compounded annually is the same as 9.53% per year compounded continuously.

Example 2: Pricing a Zero-Coupon Bond

- A zero-coupon bond pays its principal or face-value (FV) at maturity but makes no intermediate payments.

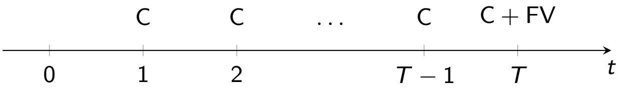


- The continuously-compounded interest rate is 8% per year.
- Consider a zero-coupon risk-free bond with face value $1,000 and expiring in seven months.
- The price of the bond is:

$$B = 1000e^{-0.08\times 7/12} = \$954.41.$$

Example 3: Pricing a Coupon-Bond

- A coupon-bond pays a periodic amount (C) either every year or every six months, and its principal or face-value (FV) at maturity.



- The continuously-compounded interest rate is 6% per year.
- Consider a bond that pays coupons of 4% every year over a notional of $1,000 and expiring in four years.
- The price of the bond is:

$$B = 40e^{-0.06\times 1} + 40e^{-0.06\times 2} + 40e^{-0.06\times 3} + 1040e^{-0.06\times 4} = \$924.65.$$

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Where Do Zero Rates Come From?

- In general, the interest rate for different maturities is not the same.
- The collection of interest rates for different maturities is called the term-structure of interest rates.
- The price of a zero-coupon bond is determined by discounting its face-value at the relevant interest rate.
- For a given maturity τ, the τ-year zero rate, denoted by r_τ, is the interest rate that gives the right τ-year zero-coupon bond price Z_τ.
- If r_τ is a continuously-compounded rate we must have:

$$Z_\tau = F e^{-r_\tau \tau}.$$

Example 4

- You have the following information for zero rates expressed per year with continuous compounding.

Maturity (months)	1	3	6	9	12
Zero Rate (%)	6.0	6.4	6.6	6.8	7.0

- Consider a zero-coupon risk-free bond with face value $1,000 and expiring in 9 months.
- The price of the bond is:

$$B = 1000e^{-0.068\times 9/12} = \$950.28.$$