The Capital Asset Pricing Model

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From Beta Pricing to the CAPM

Beta Pricing shows that the expected return of any asset A satisfies

$$\mathrm{E}(r_A) = (1 - \beta_A) r_f + \beta_A \, \mathrm{E}(r_Q)$$

for any mean-variance efficient portfolio Q. This result does not identify which portfolio is efficient, however. The Capital Asset Pricing Model (CAPM) supplies that missing link: in equilibrium, the efficient tangency portfolio Q is the **market portfolio** M.

This identification matters because constructing Q directly requires

$$\boldsymbol{w}_Q \propto \boldsymbol{\Sigma}^{-1}(\boldsymbol{\mu} - r_f \mathbf{1}).$$

In practice, both inputs are difficult to estimate. Historical average returns are noisy estimates of $\boldsymbol{\mu}$, and estimating and inverting a large covariance matrix $\boldsymbol{\Sigma}$ magnifies sampling error. Small changes in either input can therefore produce large changes in the estimated tangency weights.

Market-value weights do not require estimates of expected returns or covariances. The theoretical market portfolio nevertheless contains every risky asset in the investable universe, not just publicly listed stocks, so it is not observed exactly. In applications, investors use broad value-weighted indexes as proxies for M. The CAPM explains why the market portfolio, rather than a statistically estimated tangency portfolio, should be mean-variance efficient.

From Individual Choice to the Market Portfolio

The equilibrium argument rests on a short set of assumptions. Investors evaluate portfolios by their means and variances and share the same expected returns and covariance matrix (**homogeneous expectations**). They can trade the same assets without frictions, borrow or lend at a common risk-free rate, and choose portfolios in markets that clear. These assumptions connect individual portfolio choice to aggregate asset holdings.

Two-Fund Separation

Recall from Optimal Capital Allocation that an investor with risk aversion A who combines the risk-free asset with a risky portfolio Q chooses weight

$$w^* = \frac{\mu_Q - r_f}{A\sigma_Q^2}$$

in Q. The weight w^* depends on risk aversion, but the composition of Q does not. Because investors have homogeneous expectations, they face the same efficient frontier and select the same risky portfolio with the highest Sharpe ratio. More risk-averse investors lend and hold less of Q; less risk-averse investors may borrow and hold more of it. In either case, they hold the risky assets in the same proportions. This result is the **two-fund separation theorem**.

Aggregation and Market Clearing

Let q_j be asset j's weight in Q, and let W_i be investor i's wealth. Investor i places $w_i^* W_i q_j$ dollars in asset j. Summing across investors gives the aggregate demand for that asset:

$$q_j \sum_i w_i^* W_i.$$

The term multiplying q_j is the same for every risky asset. Consequently, the composition of aggregate risky-asset demand is exactly the composition of Q.

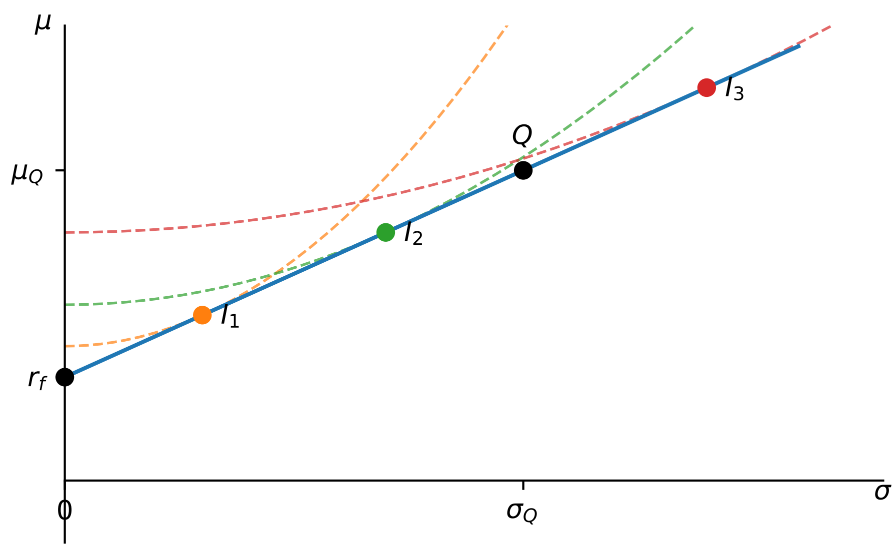

Figure 1: The figure shows three investors with different risk aversion, all holding the tangency portfolio Q combined with the risk-free asset, but in different proportions. Investor 1 lends at the risk-free rate, investor 2 is almost fully invested in Q, and investor 3 borrows to lever up her position in Q. All three hold the risky assets in exactly Q's proportions, and each investor's indifference curve (dashed) is tangent to the CAL at her chosen point, with more risk-averse investors displaying more sharply curved indifference curves.

Market clearing requires investors collectively to hold the outstanding supply of every risky asset. If V_j denotes the total market value of asset j, its weight in aggregate risky wealth is

$$\frac{V_j}{\sum_k V_k}.$$

This is also asset j's weight in the **market portfolio** M. Aggregate risky demand has Q's composition and must equal aggregate risky supply, which has M's composition. Therefore,

$$Q = M.$$

This conclusion concerns the *relative weights within the risky portfolio*. It does not require the risk-free asset to be in zero net supply. A positive net supply of borrowing or lending instruments changes the fraction of aggregate wealth invested in risky assets, but not the relative weights of those risky assets.

The assumptions are restrictive. If investors disagree about expected returns or covariances, face different borrowing rates, or cannot trade the same assets, they need not choose the same tangency portfolio. Two-fund separation then fails at the aggregate level, and market clearing no longer implies that M is mean-variance efficient.

The CAPM Pricing Equation

Property 1 (The Capital Asset Pricing Model). *If the assumptions of the CAPM hold, the market portfolio M is mean-variance efficient. The expected excess return of any asset A is therefore proportional to its market beta:*

$$\mathsf{E}(r_A) - r_f = \beta_A\big[\mathsf{E}(r_M) - r_f\big], \tag{1}$$

where

$$\beta_A = \frac{\mathsf{Cov}(r_A, r_M)}{\mathsf{V}(r_M)}$$

measures the asset's exposure to market risk.

Property 1 follows directly from the Beta Pricing proposition after equilibrium identifies Q with M. Equivalently, the CAPM expected-return equation can be written as

$$\mathrm{E}(r_A) = r_f + \beta_A\big[\mathrm{E}(r_M) - r_f\big] = (1 - \beta_A)r_f + \beta_A\,\mathrm{E}(r_M).$$

The market risk premium $\mathrm{E}(r_M) - r_f$ is the expected return investors require for bearing one unit of market risk. An asset with $\beta_A = 1$ has the same expected excess return as the market, whereas an asset with $\beta_A < 1$ has a smaller expected excess return. Idiosyncratic volatility does not enter the equation because it can be diversified away.

Example 1 (Computing a Stock Beta)**.** Suppose the correlation between stock A and the market is 0.6. If the annual standard deviation of A's return is 40% and the annual standard deviation of the market return is 20%, then

$$\begin{aligned}\beta_A &= \frac{\mathrm{Cov}(r_A, r_M)}{\mathrm{V}(r_M)} = \frac{\sigma_A \sigma_M \rho_{A,M}}{\sigma_M^2} \\ &= \frac{\sigma_A \rho_{A,M}}{\sigma_M} = \frac{0.40 \times 0.60}{0.20} = 1.2.\end{aligned}$$

A one-percentage-point change in the market return is associated with a 1.2-percentage-point change in A's return through its systematic component. □

Example 2 (Computing an Expected Return)**.** If the risk-free rate is 5%, $\beta_{DELL} = 1.3$, and $\mathrm{E}(r_M) = 14\%$, then the CAPM predicts

$$\mathrm{E}(r_{DELL}) = 0.05 + 1.3(0.14 - 0.05) = 16.7\%.$$

The 16.7% is Dell's expected annual **total return**, including both price changes and distributions such as dividends. It is an average required return, not a guarantee of the return that will be realized next year. □

Systematic and Firm-Specific Risk

Under the CAPM, an asset's return can be decomposed into a systematic component linked to the market and a firm-specific, or idiosyncratic, component:

$$r_A - r_f = \beta_A(r_M - r_f) + \varepsilon_A, \tag{2}$$

where $\mathrm{E}(\varepsilon_A) = 0$ and $\mathrm{Cov}(\varepsilon_A, r_M) = 0$. The beta term captures systematic risk, which remains in a diversified portfolio. The residual ε_A captures firm-specific risk, which diversification can reduce.

This equation also explains how beta is estimated. A regression of an asset's excess returns on the market's excess returns has slope coefficient

$$\beta_A = \frac{\mathrm{Cov}(r_A, r_M)}{\mathrm{V}(r_M)}.$$

The regression residual estimates ε_A. In empirical work, the intercept is allowed to differ from zero so that researchers can test the CAPM rather than impose it.

Because the systematic and residual components are uncorrelated, the asset's total variance is

$$\sigma_A^2 = \beta_A^2\sigma_M^2 + \sigma^2(\varepsilon_A). \tag{3}$$

The first term is systematic variance; the second is firm-specific variance. Only the exposure measured by β_A, not the amount of firm-specific variance, determines expected return under the CAPM.

Example 3 (Computing the Residual Risk)**.** Suppose stock A has an annual return standard deviation of 40%, a market beta of 1.1, and the market has an annual return standard deviation of 20%. Its residual variance is

$$\begin{aligned} \sigma^2(\varepsilon_A) &= \sigma_A^2 - \beta_A^2\sigma_M^2 \\ &= 0.40^2 - 1.1^2(0.20^2) = 0.1116. \end{aligned}$$

Therefore, the annual standard deviation of the firm-specific return is

$$\sigma(\varepsilon_A) = \sqrt{0.1116} = 33.41\%.$$

□

Interpreting Empirical Evidence

The CAPM is a statement about expected returns, which cannot be observed directly. Realized returns vary around their expectations, so a stock can earn more or less than the CAPM prediction over a particular month or year without violating the model. Empirical tests instead use many realized returns to estimate whether average returns are related to beta as the CAPM predicts.

A persistent deviation can have several interpretations. The asset may be mispriced, the market portfolio may not be mean-variance efficient because the CAPM's assumptions fail, or the value-weighted index used by the researcher may be an imperfect proxy for the theoretical market portfolio. This market-proxy problem is central to Roll (1977): because the theoretical market portfolio cannot be observed, a test based on an index also tests whether that index is an adequate proxy for M.

This ambiguity is the **joint-hypothesis problem**: every empirical test of market efficiency is also a test of the asset-pricing model used to define the correct expected return. Rejecting the CAPM relation therefore rejects the combined hypothesis, not necessarily market efficiency alone.

References

Roll, Richard. 1977. "A Critique of the Asset Pricing Theory's Tests Part I: On Past and Potential Testability of the Theory." *Journal of Financial Economics* 4 (2): 129–76.