Optimization in Python

Minimizing Functions

Main logic for the topic:

- Machine learning is fundamentally an optimization problem: we choose parameters to minimize a loss (or maximize an objective).
- Minimizing $f(x)$ is equivalent to maximizing $-f(x)$.
- Minimization and root-finding are closely connected: solving $f(x) = 0$ can be written as minimizing $f(x)^2$.
- A regression can be estimated directly ("by hand") as an optimization of squared errors, which is often useful when you want custom or more robust objective functions.
- Example:

$$f(x) = x^2 - 10x + 6.$$

- This simple convex function is only used to show how `minimize` works from different starting values.

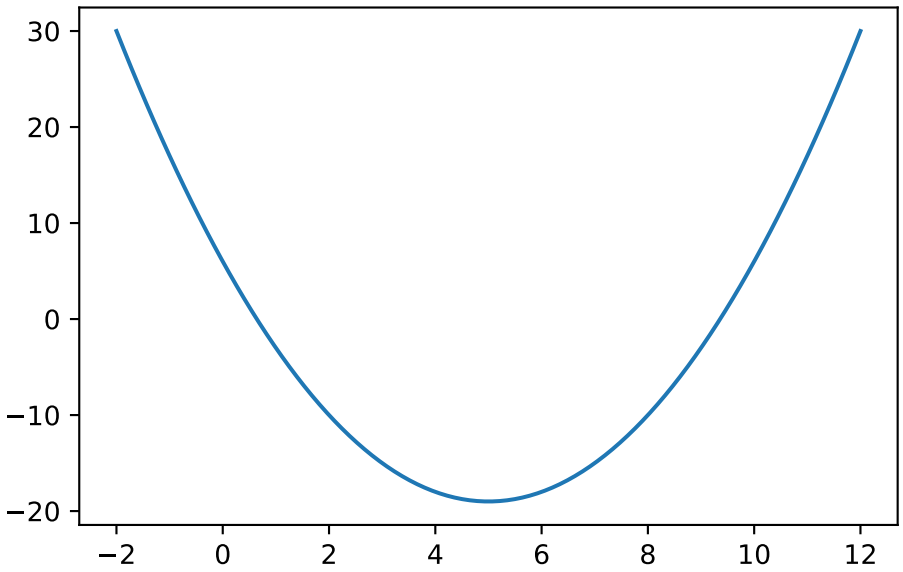
```
import numpy as np
import matplotlib.pyplot as plt
from scipy.optimize import minimize
import yfinance as yf

import warnings
warnings.simplefilter(action='ignore', category=FutureWarning)
```

```
def f(x):
    return x**2 - 10*x + 6

x = np.linspace(-2, 12, 100)
plt.plot(x, f(x))

res_left_start = minimize(f, [-100])
res_right_start = minimize(f, [100])
```



Key result:

- From start −100, optimizer converges to $x^* = 5.0$.
- From start 100, optimizer converges to $x^* = 5.0$.
- Minimum objective value is $f(x^*) = -19.0$.
- In the full output objects (`res_left_start` and `res_right_start`), `.x` is the optimizer solution, `.fun` is the objective value at the solution, and `.success` reports convergence status.

Root-Finding via Minimization

One optimization-based way to find roots of $f(x)$ is to minimize:

$$g(x) = f(x)^2.$$

If $f(x) = 0$, then $g(x) = 0$.

Why this works:

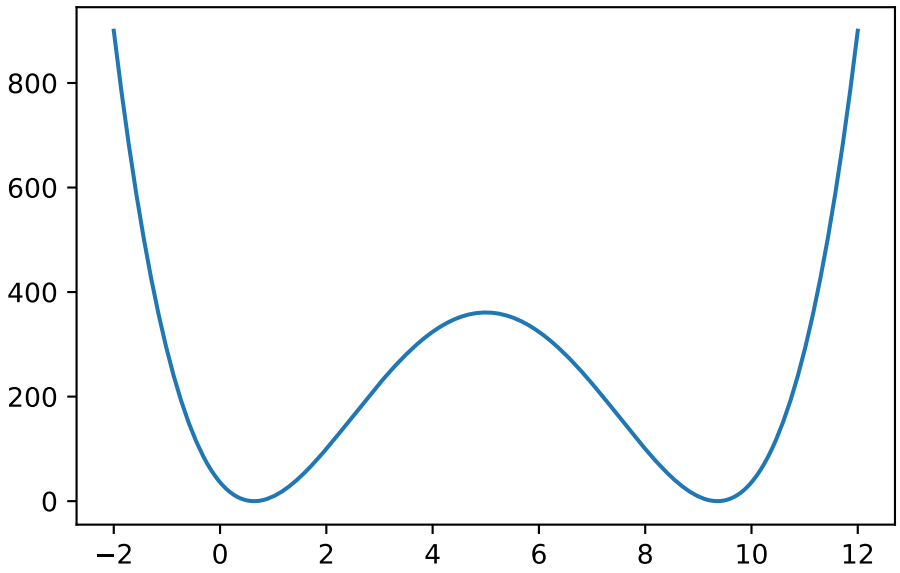
- A square is always nonnegative, so $g(x) \geq 0$ for every x.
- The minimum possible value of $g(x)$ is therefore 0.
- We get $g(x) = 0$ exactly when $f(x) = 0$, so if a root exists, any global minimizer with objective value 0 is a root.

This approach is useful as a unifying optimization view. In practice, dedicated root-finding methods can be more efficient when their assumptions are satisfied.

```
def g(x):
    return f(x)**2

x = np.linspace(-2, 12, 200)
plt.plot(x, g(x))

res_root_1 = minimize(g, [-2])
res_root_2 = minimize(g, [12])
```



Key result:

- Root near left start: $x \approx 0.64$.
- Root near right start: $x \approx 9.36$.

Regression as an Optimization Problem

Estimate a factor model by minimizing squared errors:

$$r_A = \alpha + \beta r_M + e,$$

$$\text{SSE} = \sum_i (r_{A,i} - \alpha - \beta r_{M,i})^2.$$

Interpretation:

- Standard OLS is exactly this minimization problem under squared error loss.
- In machine-learning language, this is empirical risk minimization.
- The same framework extends naturally to robust losses (for example absolute loss or Huber loss), regularization, and constraints.

- Scope note: this notebook focuses on in-sample fitting (training) of the objective; it does not perform an out-of-sample backtest.

```
df = (
    yf.download(['MSFT', 'SPY', 'QQQ'], progress=False, start='2000-01-01')['Close']
      .resample('ME')
      .last()
      .pct_change()
      .dropna()
)

def mse(theta, dep, ind):
    a, b = theta
    return np.sum((dep - a - b*ind)**2)

res_base = minimize(mse, [0, 0], args=(df.MSFT, df.SPY))
print(res_base)
```

```
  message: Optimization terminated successfully.
  success: True
   status: 0
      fun: 1.2387728751058573
        x: [ 3.175e-03  1.112e+00]
      nit: 3
      jac: [-1.490e-08  1.490e-08]
 hess_inv: [[ 1.640e-03 -6.321e-03]
            [-6.321e-03  8.450e-01]]
     nfev: 18
     njev: 6
```

Extended Models: Result Comparison

The full notebook evaluates two extensions:

- Model 1: add polynomial powers of market return.

- Model 2: add QQQ plus interaction/quadratic terms.

```
def mse1(theta, dep, ind):
    a, b1, b2, b3, b4, b5 = theta
    fit = a + b1*ind + b2*ind**2 + b3*ind**3 + b4*ind**4 + b5*ind**5
    return np.sum((dep - fit)**2)


def mse2(theta, dep, ind1, ind2):
    a, b1, b2, b3, b4, b5 = theta
    fit = a + b1*ind1 + b2*ind2 + b3*ind1*ind2 + b4*ind1**2 + b5*ind2**2
    return np.sum((dep - fit)**2)

res_poly = minimize(mse1, [0, 0, 0, 0, 0, 0], args=(df.MSFT, df.SPY))
res_two_factor = minimize(mse2, [0, 0, 0, 0, 0, 0], args=(df.MSFT, df.SPY, df.QQQ))

print(f"Base model SSE:         {res_base.fun:.4f}")
print(f"Polynomial model SSE:   {res_poly.fun:.4f}")
print(f"Two-factor model SSE:   {res_two_factor.fun:.4f}")
```

```
Base model SSE:         1.2388
Polynomial model SSE:   1.2241
Two-factor model SSE:   1.0515
```

Key results:

- Adding higher SPY powers (Model 1) barely reduces SSE relative to the base model.
- Adding QQQ with interaction and quadratic terms (Model 2) achieves a meaningful in-sample SSE reduction.

Some Final Notes

- `scipy.optimize.minimize` has many options (method choice, tolerances, constraints, bounds) that are useful in advanced problems.

- For standard linear regression workflows, `statsmodels` is usually faster and offers richer inference output.
- The optimization approach is still valuable because it extends naturally to nonlinear, constrained, or custom-loss estimation settings.