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A Market Index for Cryptocurrencies

Introduction

Main purpose in this notebook:

- Apply what we just learned about model fitting and interpretation in a real finance setting.
- Build a crypto market factor (`MKT`) and use regressions to separate common market exposure (beta) from coin-specific risk (R^2 gaps).
- Compare crypto-market returns with stock-market returns (`SPY`).

Core model: one-factor return decomposition for each coin i.

$$r_i = \alpha_i + \beta_i r_{\text{MKT}} + e_i.$$

Interpretation:

- r_{MKT}: common crypto market shock (systematic component).
- β_i: sensitivity of coin i to that common shock.
- α_i: average return component not explained by the factor.
- e_i: idiosyncratic component (coin-specific variation).

Why this model is useful:

- It decomposes returns into common risk vs coin-specific risk.
- It gives a compact way to compare coins with two summary statistics:
  - `Beta` for sensitivity.
  - R^2 for fraction of variance explained by the common factor.

Important distinction for interpretation:

- High beta does **not** necessarily imply high R^2.
- A coin can react strongly to market moves on average (high beta), yet still have large idiosyncratic volatility (low R^2).

Coins in the index:

Ticker	Coin
BTC	Bitcoin
ETH	Ethereum
BCH	Bitcoin Cash
LTC	Litecoin
XRP	Ripple
DOGE	Dogecoin
XLM	Stellar

```python
import numpy as np
import pandas as pd
import yfinance as yf
import matplotlib.pyplot as plt
import seaborn as sns
import statsmodels.formula.api as smf
import warnings

warnings.simplefilter(action='ignore', category=FutureWarning)

coins = ['BTC-USD','ETH-USD','LTC-USD','XRP-USD','DOGE-USD','BCH-USD','XLM-USD']

df = (
    yf.download(coins, start='2015-01-01', progress=False)['Close']
      .resample('ME')
      .last()
      .pct_change()
      .dropna()
)
df.columns = [c.replace('-USD','') for c in df.columns]
```

Sample period used in the code:

- Monthly returns from January 2015 onward (subject to data availability for each asset).

Building a Crypto Market Index

Factor model used for each coin i:

$$r_i = \alpha_i + \beta_i r_{\text{MKT}} + e_i.$$

`MKT` is a fixed-weight crypto index (weights based on market-cap shares).

Modeling choice:

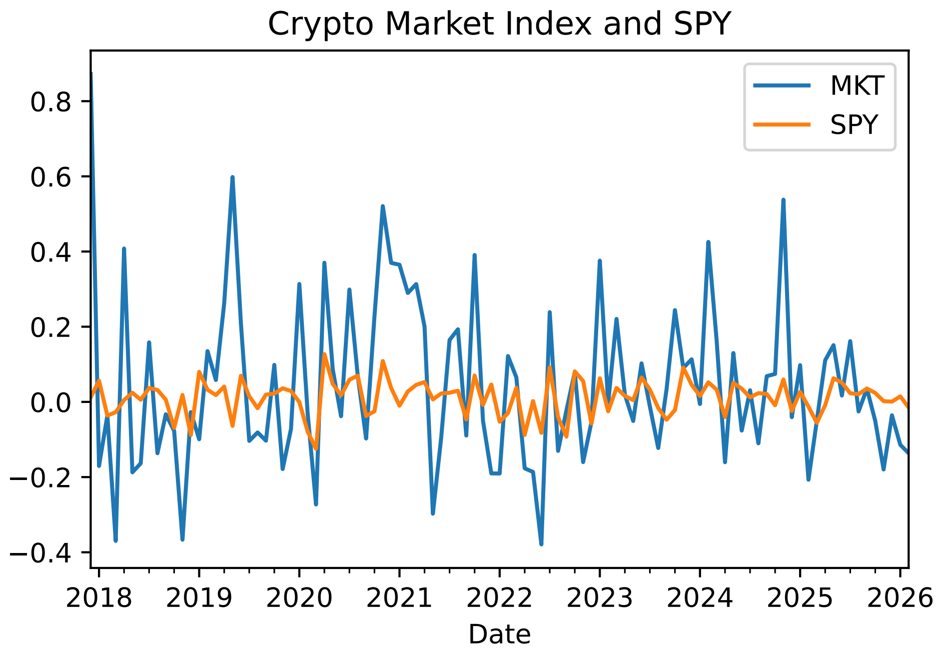
- We use fixed weights for transparency and stability of interpretation.
- With time-varying weights, measured beta can mix true exposure changes with index-reweighting effects.

```
w_raw = pd.Series({
    'BTC': 57.5,
    'ETH': 11.6,
    'XRP': 3.8,
    'DOGE': 0.7,
    'BCH': 0.4,
    'LTC': 0.2,
    'XLM': 0.2
})

w = w_raw / w_raw.sum()
weights = pd.DataFrame(np.tile(w.values, (len(df.index), 1)), index=df.index, columns=w.inde
df['MKT'] = (weights * df[w.index]).sum(axis=1)
```

Analyzing the Relationship Between Cryptocurrencies and the Crypto Market Index

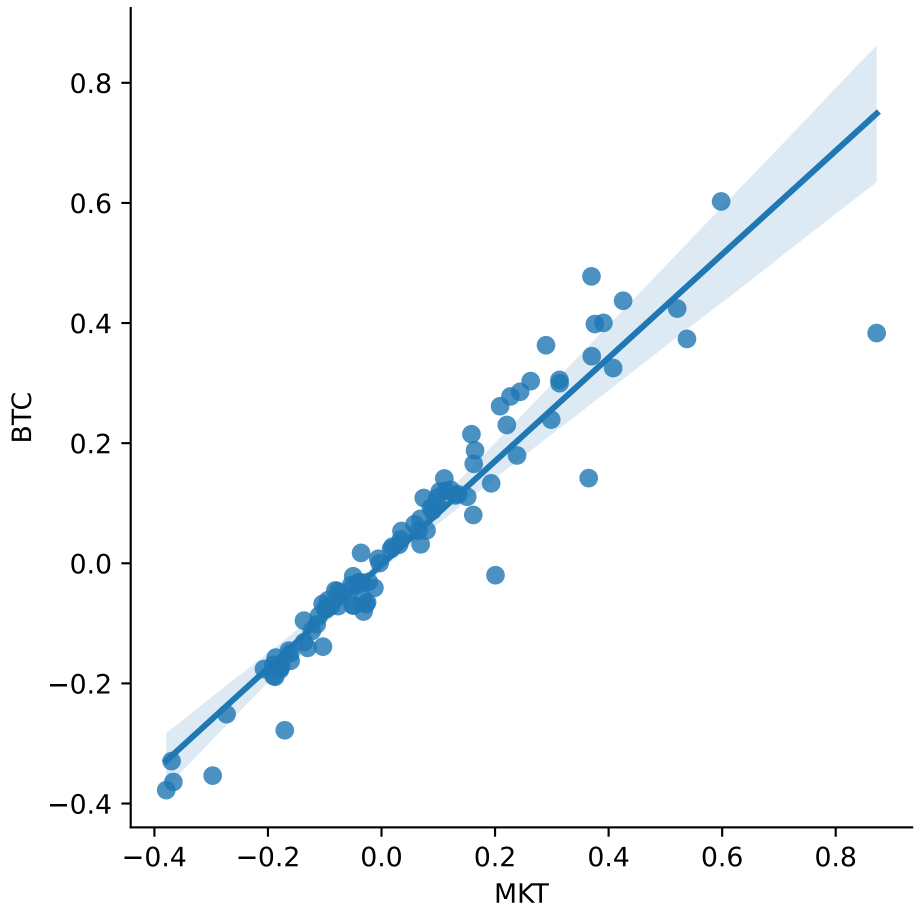
First, we benchmark crypto market volatility against equities:



Key results (in this sample):

- The crypto market index is much more volatile than SPY.

BTC vs MKT

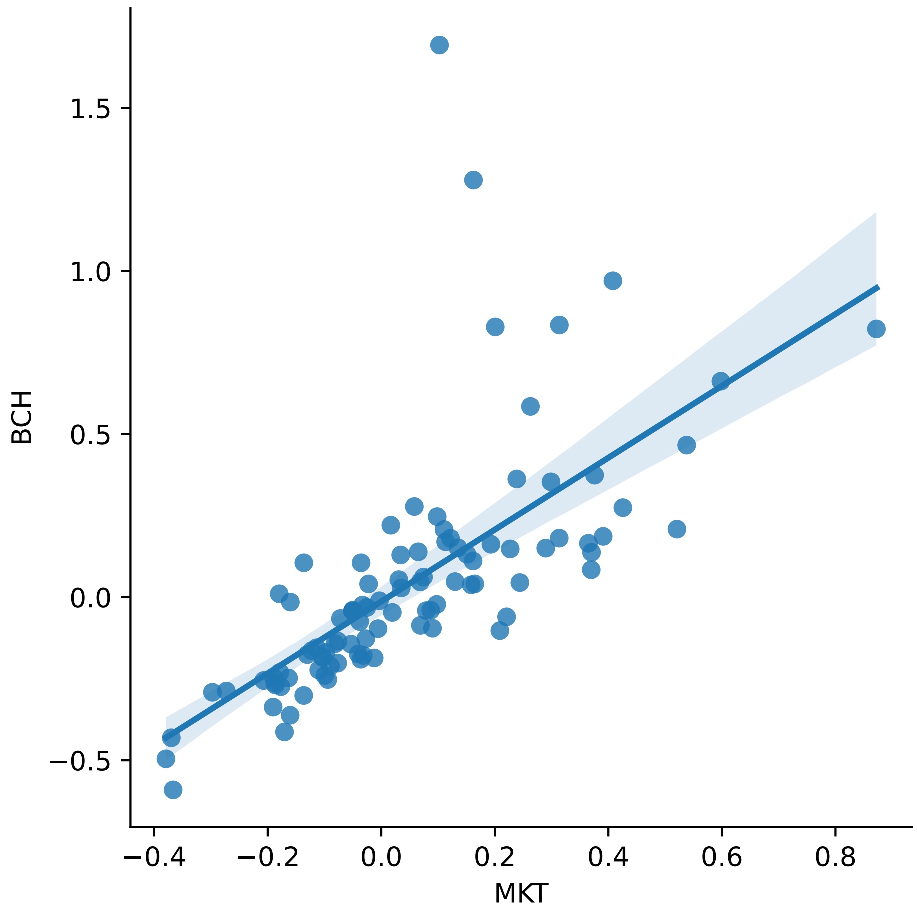


	beta_MKT	p_value(beta=0)	R_squared
0	0.8613	0.0	0.9023

BTC interpretation:

- BTC has strong positive exposure to the crypto market factor.
- The fit is tight (high R^2), consistent with BTC's large index weight.

BCH vs MKT



	beta_MKT	p_value(beta=0)	R_squared
0	1.1017	0.0	0.4747

BCH interpretation:

- BCH still has meaningful market exposure.
- The relationship is noisier than BTC (lower R^2), so more variation is coin-specific.

All Coins on MKT

	Coin	Beta	R-squared
7	MKT	1.00	1.000
1	BTC	0.86	0.902
3	ETH	1.08	0.721
4	LTC	1.06	0.690
0	BCH	1.10	0.475
5	XLM	2.07	0.432
6	XRP	2.59	0.368
2	DOGE	2.13	0.229

Cross-coin interpretation:

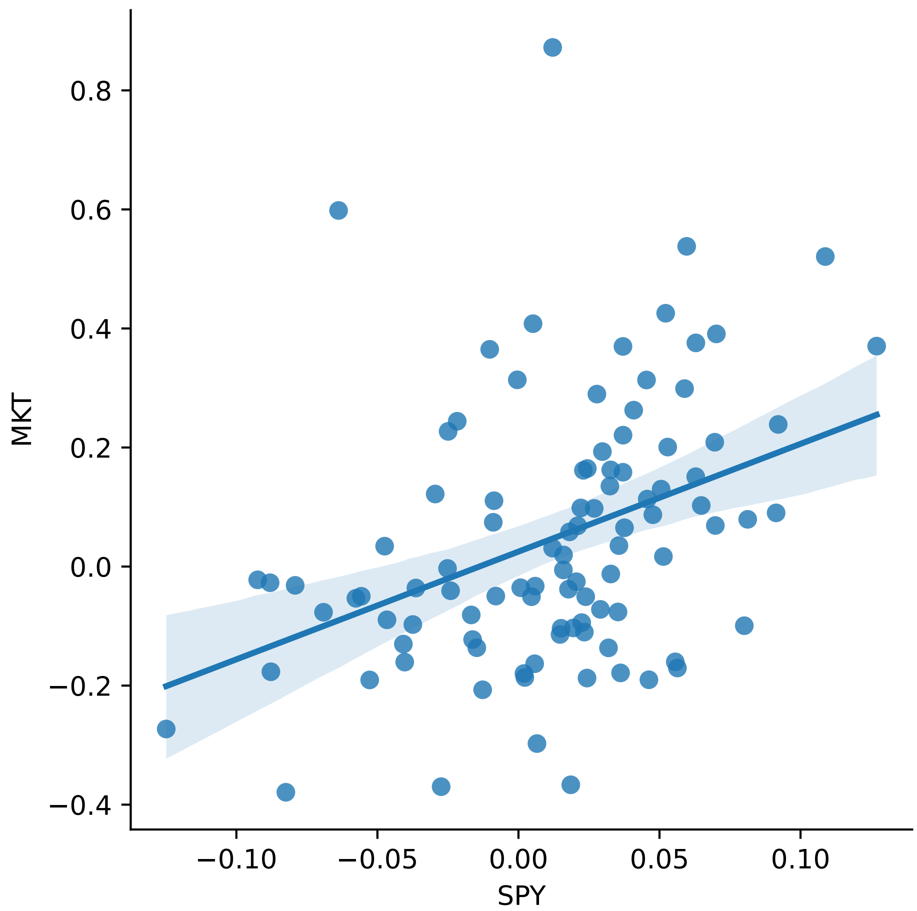
- High beta with low R^2 means strong average market sensitivity but substantial idiosyncratic risk.
- DOGE, XLM, and XRP show relatively high betas but lower R^2, indicating strong average market sensitivity alongside substantial coin-specific risk.
- Sanity check: the `MKT` row should show beta $= 1$ and $R^2 = 1$ when regressing `MKT` on itself.

Statistical reading guide:

- Testing $\beta = 0$: asks whether market exposure is statistically different from zero.
- Confidence intervals: show plausible ranges for true exposure.
- Economic significance (beta size) and statistical significance (p-value) are related but distinct.

Crypto Market Index vs. Stock Market

Scatter relationship:



	beta_SPY	p_value(beta_SPY=0)	p_value(beta_SPY=1)	R_squared
0	1.8102	0.0001	0.0682	0.1489

Key results (in this sample):

- `SPY` loading in `MKT ~ SPY` is positive and statistically significant (reject $\beta_{SPY} = 0$).
- Testing $\beta_{SPY} = 1$: at the 5% level we fail to reject equality to 1, though the result is borderline at the 10% level.

Economic interpretation:

- A positive, significant SPY loading indicates directional co-movement between crypto and equities.

- But R^2 is still crucial: co-movement can be significant while a large share of crypto variation remains unexplained by equities.

Takeaways

- This notebook applies the same regression logic from previous notes to a realistic asset-pricing use case.
- `Beta` measures average market sensitivity; R^2 measures how much variation the market factor explains.
- Crypto and equities co-move, but a large share of crypto risk remains crypto-specific.
- Scope note: this notebook is an in-sample modeling exercise (fit and interpretation), not a trading backtest.