Decentralized Exchanges and AMMs

Introduction

Main purpose in this notebook:

- Explain constant-product AMM pricing.
- Quantify slippage, arbitrage alignment, and LP tradeoffs.
- Connect AMM mechanics to standard market-microstructure logic.

Market Structure and Motivation

Core contrast:

- CEXs mainly use order books (CLOB-style matching).
- AMMs execute against pooled reserves and formula-based quotes.

In AMMs, the pool is the immediate counterparty, and price updates through reserve changes (Adams et al. 2020).

Constant-Product Pricing

Baseline AMM invariant:

$$xy = k.$$

Marginal quote (in y per unit of x):

$$P = \frac{y}{x}.$$

```python
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt

x0 = 1_000.0
y0 = 1_000.0
k = x0 * y0

def cp_swap_buy_x(delta_y, x=x0, y=y0):
    y_new = y + delta_y
    x_new = k / y_new
    dx_out = x - x_new
    avg_price = delta_y / dx_out
    return {
        "delta_y_in": delta_y,
        "x_out": dx_out,
        "avg_price": avg_price,
        "marginal_before": y / x,
        "marginal_after": y_new / x_new,
    }

trades = pd.DataFrame([cp_swap_buy_x(dy) for dy in [10, 50, 100, 200, 400]])
trades.round(4)
```

	delta_y_in	x_out	avg_price	marginal_before	marginal_after
0	10	9.9010	1.01	1.0	1.0201
1	50	47.6190	1.05	1.0	1.1025
2	100	90.9091	1.10	1.0	1.2100
3	200	166.6667	1.20	1.0	1.4400
4	400	285.7143	1.40	1.0	1.9600

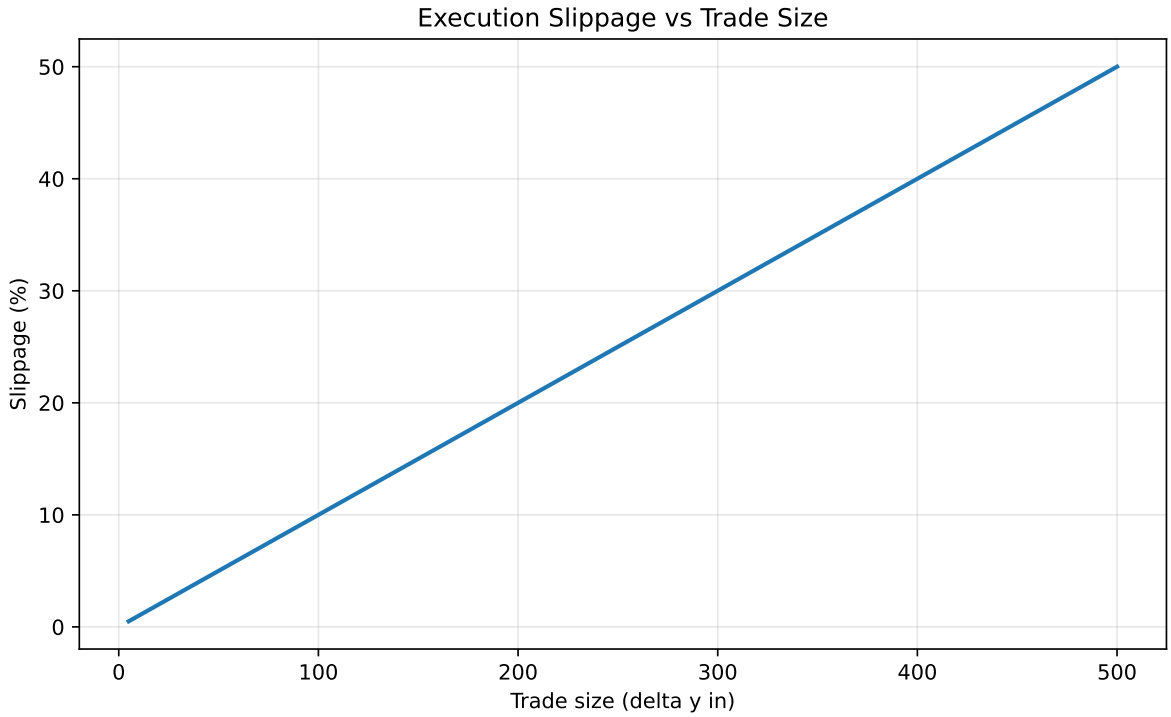
Key interpretation:

- Larger trades move price more (endogenous impact), analogous to walking the book.

Slippage and Trade Size

Slippage metric used in the notebook:

$$\text{slippage} = \frac{P_{\text{avg}}}{P_{\text{pre}}} - 1.$$



Key result:

- Slippage rises nonlinearly with trade size relative to pool depth.

Arbitrage and Price Alignment

When external price is P^*, no-fee aligned reserves satisfy:

$$\frac{y}{x} = P^*, \quad xy = k,$$

so

$$x = \sqrt{\frac{k}{P^*}}, \qquad y = \sqrt{kP^*}.$$

	p_external	x_reserve	y_reserve	pool_price
0	0.8	1118.0340	894.4272	0.8
1	1.0	1000.0000	1000.0000	1.0
2	1.2	912.8709	1095.4451	1.2
3	1.5	816.4966	1224.7449	1.5

Key interpretation:

- AMM prices adjust via arbitrage flow; reserves transmit external information into on-chain prices (Capponi et al. 2026).

LP Fees vs Impermanent Loss

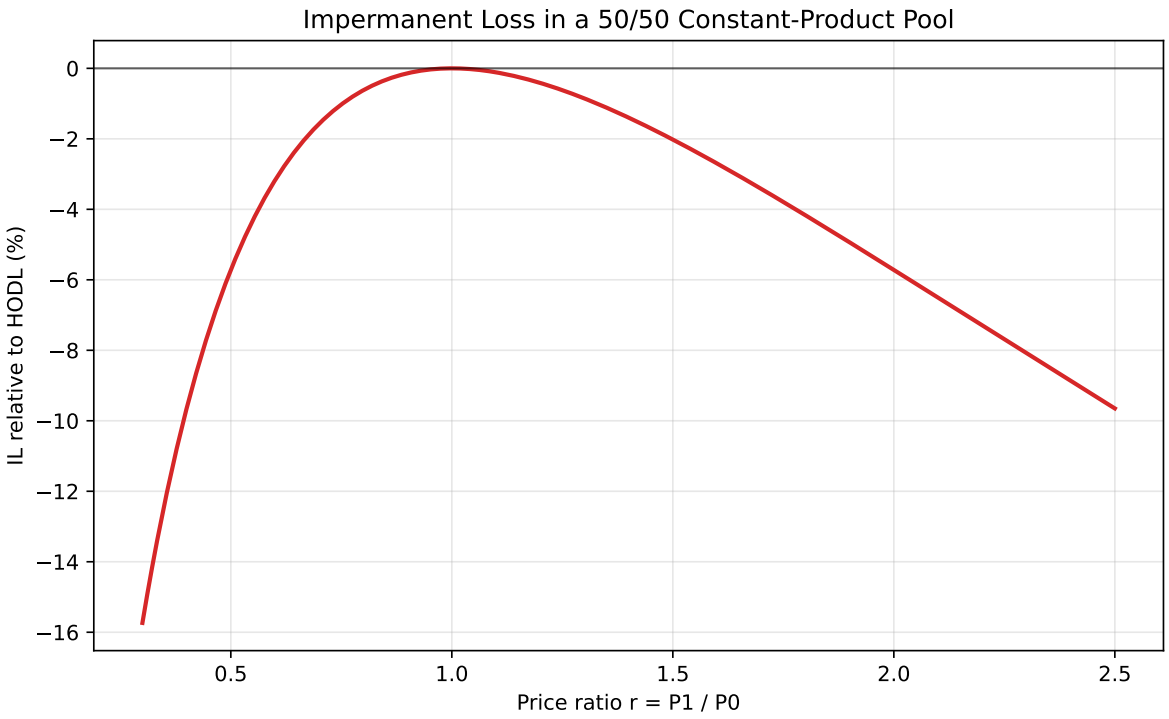
Impermanent loss (IL) benchmark in a 50/50 pool:

$$\text{IL}(r) = \frac{2\sqrt{r}}{1+r} - 1, \qquad r = \frac{P_1}{P_0}.$$

Fee income benchmark:

$$\text{LP fee income} \approx s\, f\, V,$$

where s is LP share, f fee rate, V traded notional.



Key interpretation:

- LP outcomes depend on fee income versus volatility-driven inventory drift (IL) (Capponi et al. 2025).

How Uniswap v3 and v4 Differ from This Notebook

Notebook baseline is v2-style full-range liquidity.

- v3 adds concentrated liquidity (range choice, path-dependent fee earning).
- v4 keeps concentration and adds programmable hooks/singleton architecture (Uniswap Labs 2025; Adams et al. 2024).

Takeaways

- AMMs provide transparent, rule-based market making.
- Three central objects for LP outcomes: depth, volume, volatility.

- Arbitrage aligns prices, but LPs still bear inventory and implementation risk.
- The notebook is a microstructure mechanics exercise, not a full production risk model.

Adams, Hayden, Noah Zinsmeister, Moody Salem, et al. 2024. *Uniswap V4 Core*. Uniswap v4 Whitepaper. https://app.uniswap.org/whitepaper-v4.pdf.

Adams, Hayden, Noah Zinsmeister, Moody Salem, River Keefer, and Dan Robinson. 2020. *Uniswap V2 Core*. Uniswap Whitepaper. https://docs.uniswap.org/whitepaper.pdf.

Capponi, Agostino, Ruizhe Jia, Yubo Ma, John Wang, and Boyu Zhu. 2025. "Liquidity Provision on Blockchain-Based Decentralized Exchanges." *Review of Financial Studies* 38 (10): 3040–85. https://doi.org/10.1093/rfs/hhaf046.

Capponi, Agostino, Ruizhe Jia, and Shuo Yu. 2026. "Price Discovery on Decentralized Exchanges." *Review of Financial Studies*, ahead of print. https://doi.org/10.1093/rfs/hhag002.

Uniswap Labs. 2025. *Uniswap V4 Is Here*. https://blog.uniswap.org/uniswap-v4-is-here.