

## The Law of One Price and the SDF

This notebook introduces the **stochastic discount factor** (SDF), one of the central objects in asset pricing. The SDF is a random variable  $m$  that prices every traded payoff through the formula  $p(x) = E(mx)$ . Its importance is that it compresses all of asset pricing into a single object: once  $m$  is known, we can value any traded cash flow, recover the risk-free rate, and decompose expected returns into time-value-of-money and risk-premium components. The same idea also creates a bridge between no-arbitrage and equilibrium theory. In no-arbitrage language, the SDF is the linear functional that prices payoffs consistently across assets; in equilibrium language, it is the marginal value of wealth or consumption across states, as in Lucas (1978) and Breeden (1979). This is why the SDF, or **pricing kernel**, is the common language of modern asset pricing; see also Cochrane (2009).

Rather than taking the pricing formula as given, we derive it from the **law of one price** alone, which implies that prices are generated by a linear pricing rule on the space of traded payoffs. That is enough to obtain an SDF, the risk-free rate, risk premia, and the Hansen-Jagannathan bound, and to show that the mean-variance frontier of the [portfolio frontier notebooks](#) and the SDF are two descriptions of the same object. What the law of one price does not deliver is a positive SDF. The [next notebook](#) adds the principle of no-arbitrage for that. The representation used here follows the SDF approach developed in Hansen and Richard (1987), Hansen and Jagannathan (1991), and Hansen and Jagannathan (1997).

## The Set of Traded Payoffs

Let  $L = \mathbb{R}^S$  denote the full space of state-contingent payoffs: each element of  $L$  specifies one payoff in each of the  $S$  states of the world. In this economy, not every payoff in  $L$  is necessarily traded unless the market is *complete*. In a finite-state economy, we identify each payoff with its vector of state realizations in  $\mathbb{R}^S$ . We use the following bracket convention throughout. Square

brackets display the state-by-state realizations of a random variable or any vector indexed by states; for example, if  $u, v \in \mathbb{R}^S$ , then

$$u = \begin{bmatrix} u(1) \\ \vdots \\ u(S) \end{bmatrix}, \quad \text{and} \quad v = \begin{bmatrix} v(1) \\ \vdots \\ v(S) \end{bmatrix},$$

and the inner product of two vectors in  $\mathbb{R}^S$  is written  $u \cdot v = \sum_{s=1}^S u(s) v(s)$ . A prime ( $'$ ) denotes transposition.

Let  $x_1, x_2, \dots, x_N$  denote linearly independent traded payoffs that form a basis for the marketed payoff space

$$X = \text{span}\{x_1, x_2, \dots, x_N\},$$

where  $N = \dim(X) \leq S$ . Here  $\mathbb{R}^N$  is the coordinate space for portfolios and other stacked traded-asset objects. Vectors in this asset space will typically be written with decorated symbols such as  $\mathbf{x}$  or  $\boldsymbol{\pi}$ ; when the dimension is clear from context, we use the same dot-product notation in either space. Round parentheses stack objects of the same kind into a structural column vector in asset space. Denote by

$$\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \end{pmatrix} \tag{1}$$

a column vector containing all the basis payoffs. The associated Gram matrix

$$E(\mathbf{x}\mathbf{x}') = \sum_{s=1}^S q(s) \mathbf{x}(s) \mathbf{x}'(s)$$

is invertible because the payoffs  $x_1, \dots, x_N$  are linearly independent.

We can create other payoffs by buying or selling portfolios of these  $N$  traded payoffs. Any  $x \in X$  can be expressed as:

$$x = \sum_{i=1}^N a_i x_i, \tag{2}$$

for  $a_i \in \mathbb{R}$ ,  $1 \leq i \leq N$ . Finally, we denote by  $\pi_i$  the price of traded payoff  $x_i$  for  $1 \leq i \leq N$ ,

and by

$$\boldsymbol{\pi} = \begin{pmatrix} \pi_1 \\ \pi_2 \\ \vdots \\ \pi_N \end{pmatrix}$$

a column vector containing the prices of the  $N$  basis payoffs.

## The Law of One Price

Now, we would like to know if there is a way to create a pricing functional  $p : X \rightarrow \mathbb{R}$  that gives the price of any traded payoff. Clearly, we have that  $p(x_i) = \pi_i$  for  $1 \leq i \leq N$ , and our intuition tells us that the price of any other traded payoff should be pinned down by the prices of the basis assets used to construct it. The next example shows what goes wrong when this consistency fails.

**Example 1.** Suppose that  $p(x) = 1$  and  $p(y) = 2$ . There is also an asset  $z = 3x + 4y$  such that  $p(z) = 12$ . Is there an arbitrage opportunity?

Of course! We could buy 3 units of  $x$  and 4 units of  $y$  and bundle them as  $z$ . The cost of the bundle is \$11, but we can sell it for \$12, generating a riskless profit of \$1 per trade. Since there is no shortage of these securities, we could continue doing it until the prices of  $x$  and  $y$  go up and/or the price of  $z$  goes down. □

In order to avoid this type of situation, we will assume the following.

**Assumption 1** (The Law of One Price). *Suppose that  $x_i \in X$  and  $a_i \in \mathbb{R}$  for  $i \in \{1, 2, \dots, N\}$ . If  $x = \sum_{i=1}^N a_i x_i \in X$ , then*

$$p(x) = \sum_{i=1}^N a_i p(x_i). \tag{3}$$

In competitive markets, the **law of one price** (LOOP) guarantees that the price of a basket of stocks is equal to the sum of the prices of its constituents. This logic is at the heart of how **Exchange-Traded Funds** (ETF) operate, as the next example shows.

**Example 2.** An ETF is a type of investment fund that is traded on stock exchanges, similar to individual stocks. ETFs hold a diversified portfolio of assets, such as stocks, bonds, or commodities, which provides investors with broad exposure to specific markets or investment strategies.

ETF arbitrage is the mechanism that helps keep the market price of an ETF in line with its Net Asset Value (NAV). Authorized Participants (APs), typically large financial institutions, have the ability to create or redeem ETF shares in large blocks called creation units.

When the ETF market price is higher than the NAV, APs can buy the underlying securities of the ETF in the open market and then deliver them to the ETF issuer in exchange for new ETF shares. The AP can then sell these ETF shares at the higher market price, making a profit. This buying of underlying securities pushes their prices up, while the selling of new ETF shares pushes the ETF price down, bringing the two prices closer together.

When the ETF market price is lower than the NAV, APs can buy ETF shares in the open market and deliver them to the ETF issuer in exchange for the underlying securities. The AP can then sell these underlying securities at the higher NAV price, making a profit. This buying of ETF shares pushes their price up, while the selling of the underlying securities pushes their prices down, again bringing the two prices closer together.

This creation and redemption process happens continuously and helps to keep the ETF price in line with the NAV. The arbitrage opportunities are typically small but sufficient for APs to engage in the process for profit, ensuring that the ETF price does not deviate significantly from its NAV.

You can find more information [here](#). □

The fact that the market for ETFs is so liquid and works flawlessly reassures us that LOOP is a reasonable axiom to start working from. The law of one price implies that the price functional defined in (3) is linear on the finite-dimensional payoff space  $X$ , hence continuous. By the [Riesz Representation Theorem](#), there exists a unique  $x^* \in X$  such that  $p(x) = E(x^*x)$  for all  $x \in X$ .

Since  $p(x_i) = \pi_i$  for each basis asset, the representing payoff  $x^*$  itself satisfies  $E(x^*\mathbf{x}) = \boldsymbol{\pi}$  and is therefore a valid SDF. More generally, if  $m$  is any valid SDF, then  $E(m\mathbf{x}) = \boldsymbol{\pi}$ . Substituting

into the [Projection onto a Subspace](#) formula (with  $y = m$ ,  $M = X$ , and  $G = E(\mathbf{x}\mathbf{x}')$ ) identifies  $x^*$  as the projection of any valid SDF onto  $X$ .

**Property 1.** *Under LOOP, the unique SDF in  $X$  is the projection of any valid SDF  $m$  onto  $X$ , given by*

$$x^* = \boldsymbol{\pi}' E(\mathbf{x}\mathbf{x}')^{-1} \mathbf{x} \in X. \quad (4)$$

To verify directly, take any  $x = \mathbf{x}'\mathbf{a} \in X$ , where  $\mathbf{a} = (a_1, \dots, a_N)'$ , and compute

$$\begin{aligned} E(x^*x) &= E(\boldsymbol{\pi}' E(\mathbf{x}\mathbf{x}')^{-1} \mathbf{x}\mathbf{x}'\mathbf{a}) \\ &= \boldsymbol{\pi}' E(\mathbf{x}\mathbf{x}')^{-1} E(\mathbf{x}\mathbf{x}')\mathbf{a} \\ &= \boldsymbol{\pi}'\mathbf{a} = p(x), \end{aligned}$$

confirming that  $x^*$  prices all assets correctly.

**Example 3.** As a concrete example, suppose there are four states with equal probabilities  $P(\omega) = 0.25$  and two traded payoffs

$$x_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \quad x_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 2 \end{bmatrix}.$$

Collect them in the column vector

$$\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}.$$

So  $X = \text{span}\{x_1, x_2\}$ . Take a valid SDF

$$m = \begin{bmatrix} 1.2 \\ 0.8 \\ 1.0 \\ 0.6 \end{bmatrix},$$

which need not belong to  $X$ . The corresponding asset prices are

$$\boldsymbol{\pi} = E(m\mathbf{x}) = \begin{pmatrix} E(mx_1) \\ E(mx_2) \end{pmatrix} = \begin{pmatrix} 0.5 \\ 0.75 \end{pmatrix}.$$

Also,

$$E(\mathbf{x}\mathbf{x}') = \begin{pmatrix} E(x_1^2) & E(x_1x_2) \\ E(x_1x_2) & E(x_2^2) \end{pmatrix} = \begin{pmatrix} 0.5 & 0.25 \\ 0.25 & 1.5 \end{pmatrix}.$$

Applying equation (4),

$$\mathbf{x}^* = \boldsymbol{\pi}' E(\mathbf{x}\mathbf{x}')^{-1} \mathbf{x} = \frac{9}{11}x_1 + \frac{4}{11}x_2.$$

This  $\mathbf{x}^*$  lies in the payoff space  $X$  and satisfies

$$E(x^*x_1) = 0.5, \quad E(x^*x_2) = 0.75,$$

so it prices the traded assets exactly even though the original SDF  $m$  is not itself in  $X$ . □

Any other SDF  $m$  will price the assets correctly, so that

$$E((m - x^*)x) = E(mx) - E(x^*x) = p(x) - p(x) = 0.$$

This shows that we can create new SDFs by combining  $x^*$  with any random variable  $\varepsilon$  orthogonal to  $X$ . In other words, all the SDFs that price assets correctly in  $X$  can be written as:

$$m = x^* + \varepsilon,$$

where  $\varepsilon \perp x$  for all  $x \in X$ .

**Example 4.** Continuing Example 3, the orthogonal component is

$$\varepsilon = m - x^* = \begin{bmatrix} \frac{21}{55} \\ \frac{21}{55} \\ -\frac{7}{55} \\ \frac{11}{55} \\ -\frac{11}{55} \end{bmatrix}.$$

Using the equal state probabilities, we obtain

$$E(\varepsilon x_1) = 0.25 \left( \frac{21}{55} - \frac{21}{55} \right) = 0$$

and

$$E(\varepsilon x_2) = 0.25 \left( -\frac{21}{55} + \frac{7}{11} - \frac{14}{55} \right) = 0.$$

Therefore, for any traded payoff  $x = a_1 x_1 + a_2 x_2 \in X$ ,

$$E(x\varepsilon) = a_1 E(x_1\varepsilon) + a_2 E(x_2\varepsilon) = 0,$$

which verifies in this example that  $\varepsilon \in X^\perp$ . □

## Arrow-Debreu Securities

Arrow-Debreu securities are the most elementary financial contracts imaginable: security  $e_s$  pays \$1 if state  $s$  occurs and \$0 in every other state. They are the atomic building blocks of the full payoff space  $L$ : every contingent claim can be decomposed state by state using these elementary random variables, whether or not the corresponding securities are actually traded.

Formally, define the random variable  $e_s$  by

$$e_s(j) = \begin{cases} 1 & \text{if } j = s, \\ 0 & \text{otherwise,} \end{cases}$$

for each  $s \in \{1, 2, \dots, S\}$ . Since the payoff space  $L$  has dimension  $S$ , the collection  $\{e_1, e_2, \dots, e_S\}$  forms a basis for  $L$ . Any payoff  $x \in L$  therefore admits the unique decomposition

$$x = x(1) e_1 + x(2) e_2 + \dots + x(S) e_S, \tag{5}$$

since in state  $s$  the right-hand side evaluates to  $x(s) 1 = x(s)$ .

This decomposition is purely geometric. It does not mean that every Arrow-Debreu security is traded. When  $N < S$ , the marketed payoff space  $X$  is only a subspace of  $L$ , so many contingent

claims in  $L$  need not be attainable by portfolios of traded assets.

If an Arrow-Debreu security  $e_s$  is traded, its price is called the **state price** for state  $s$  and is denoted  $\psi(s)$ . In a complete market, where all  $S$  Arrow-Debreu securities are traded, the price of any payoff  $x \in L$  follows immediately from the spanning representation:

$$p(x) = \sum_{s=1}^S \psi(s) x(s) = \psi \cdot x,$$

where

$$\psi = \begin{bmatrix} \psi(1) \\ \vdots \\ \psi(S) \end{bmatrix}$$

collects all state prices. In that case prices are inner products with the state-price vector.

More generally, even when Arrow-Debreu securities are not traded individually, any SDF  $m$  induces a *state-price vector*

$$\psi(s) = q(s) m(s),$$

and the pricing equation becomes

$$p(x) = E(mx) = \sum_{s=1}^S q(s) m(s) x(s) = \sum_{s=1}^S \psi(s) x(s) = \psi \cdot x.$$

Thus an SDF and a state-price vector are just two representations of the same linear pricing rule: the SDF  $m$  weights payoffs by their probability-adjusted marginal value, while the state-price vector  $\psi$  absorbs the probabilities directly, so that  $p(x) = E(mx) = \psi \cdot x$  holds in either form. The two are linked by the change of variables  $\psi(s) = q(s) m(s)$ , which is invertible whenever  $q(s) > 0$ . The positivity condition  $m > 0$  is therefore equivalent to  $\psi(s) > 0$  for all  $s$ , the condition the [next notebook](#) ties to the absence of arbitrage.

**Example 5.** Continuing Example 3, the original SDF

$$m = \begin{bmatrix} 1.2 \\ 0.8 \\ 1.0 \\ 0.6 \end{bmatrix}$$

induces the state-price vector

$$\psi^m = \frac{1}{4}m = \begin{bmatrix} 0.3 \\ 0.2 \\ 0.25 \\ 0.15 \end{bmatrix}.$$

The projected SDF

$$x^* = \frac{9}{11}x_1 + \frac{4}{11}x_2$$

induces instead

$$\psi^* = \frac{1}{4}x^* = \begin{bmatrix} \frac{9}{44} \\ \frac{13}{44} \\ \frac{1}{11} \\ \frac{2}{11} \end{bmatrix}.$$

These two state-price vectors are different because the market is incomplete, but they agree on traded assets:

$$p(x_1) = \psi^m \cdot x_1 = 0.3 + 0.2 = 0.5,$$

$$p(x_2) = \psi^m \cdot x_2 = 0.2 + 0.25 + 2(0.15) = 0.75,$$

and

$$p(x_1) = \psi^* \cdot x_1 = \frac{9}{44} + \frac{13}{44} = 0.5,$$

$$p(x_2) = \psi^* \cdot x_2 = \frac{13}{44} + \frac{1}{11} + 2\left(\frac{2}{11}\right) = 0.75.$$

So both  $\psi^m$  and  $\psi^*$  represent valid pricing rules for the traded payoff space  $X$ . □

If all  $S$  Arrow-Debreu securities are traded, so that  $N = S$  and  $X = L$ , the market is **complete**. In this case the state-price vector  $\psi$  is uniquely determined by the  $S$  asset prices, and so is the SDF:  $m(s) = \psi(s)/q(s)$  for each  $s$ . Incomplete markets ( $N < S$ ) admit multiple valid state-price vectors and hence multiple SDFs, all of which price the  $N$  traded assets correctly

but assign different values to non-traded payoffs.

## Returns and the Risk-Free Rate

We now translate the pricing relation into a restriction on expected returns. For any traded payoff  $x$  with non-zero price  $p(x) \neq 0$ , define its gross return by  $R = x/p(x)$ . We work with gross returns throughout this section.

Since  $p(x) = E(mx)$ , dividing both sides by  $p(x)$  gives

$$1 = E(mR).$$

Thus, every traded return has discounted expectation equal to one under any valid SDF.

Assume now there is a traded risk-free asset that pays 1 in every state of the world. Its price satisfies

$$p(1) = E(m),$$

for any valid SDF  $m$ . Therefore  $E(m)$  is the same for all SDFs, because they must all assign the same price to the same traded payoff. We assume  $p(1) > 0$ , so that its return is defined. The [next notebook](#) shows that no-arbitrage guarantees it. The gross return of this asset, which is constant in all states of the world, is the risk-free rate

$$R^f = \frac{1}{p(1)} = \frac{1}{E(m)}.$$

Thus, the risk-free rate is the reciprocal of the expected SDF. A higher average discount factor implies a lower risk-free rate.

For any traded asset  $i$ ,

$$1 = E(mR^i) = E(m) E(R^i) + \text{Cov}(m, R^i).$$

Substituting  $E(m) = 1/R^f$  and rearranging:

$$E(R^i) - R^f = -\frac{\text{Cov}(m, R^i)}{E(m)} = -R^f \text{Cov}(m, R^i). \quad (6)$$

The excess return of any asset equals  $-R^f$  times its covariance with the SDF. Assets that tend to pay off when the SDF is high, that is, when the marginal value of wealth is high, typically in bad times, are valuable as insurance and therefore command lower excess returns. Assets that pay off when the SDF is low, in good times, offer little insurance and must compensate investors with higher expected returns.

## The Hansen-Jagannathan Bound

The **Sharpe ratio** of asset  $i$  is its excess expected return per unit of return volatility:

$$SR^i = \frac{E(R^i) - R^f}{\sigma(R^i)},$$

provided  $\sigma(R^i) > 0$ . It is the most widely used measure of risk-adjusted performance. We will show that any valid SDF places a universal upper bound on the Sharpe ratio of every traded asset.

Writing the covariance in equation (6) in terms of the correlation  $\rho_{m,i}$ ,

$$\begin{aligned} E(R^i) - R^f &= -\frac{\text{Cov}(m, R^i)}{E(m)} \\ &= -\frac{\rho_{m,i} \sigma(m) \sigma(R^i)}{E(m)}. \end{aligned}$$

Dividing by  $\sigma(R^i)$  and taking absolute values gives

$$\left| \frac{E(R^i) - R^f}{\sigma(R^i)} \right| = |\rho_{m,i}| \frac{\sigma(m)}{E(m)}.$$

Since the [Cauchy-Schwarz inequality](#) implies  $|\rho_{m,i}| \leq 1$ , we obtain

$$\left| \frac{E(R^i) - R^f}{\sigma(R^i)} \right| \leq \frac{\sigma(m)}{E(m)}. \quad (7)$$

This is the **Hansen-Jagannathan bound** (Hansen and Jagannathan 1991): the absolute Sharpe

ratio of any traded asset is bounded above by the coefficient of variation  $\sigma(m)/E(m)$  of the SDF. The bound holds for every valid SDF  $m$  and every traded asset  $i$  with non-zero return volatility. Equality holds when  $|\rho_{m,i}| = 1$ , i.e., when the return is perfectly correlated with the SDF up to sign.

Taking the supremum over all traded returns with non-zero volatility gives

$$\sup_{R \in \mathcal{R}} \left| \frac{E(R) - R^f}{\sigma(R)} \right| \leq \frac{\sigma(m)}{E(m)},$$

where  $\mathcal{R}$  is the set of traded gross returns. Since this inequality holds for every valid SDF, taking the infimum over the set  $\mathcal{M}$  of valid SDFs yields

$$\sup_{R \in \mathcal{R}} \left| \frac{E(R) - R^f}{\sigma(R)} \right| \leq \inf_{m \in \mathcal{M}} \frac{\sigma(m)}{E(m)}.$$

In fact, Cauchy-Schwarz is sharp, so

$$\sup_{R \in \mathcal{R}} \left| \frac{E(R) - R^f}{\sigma(R)} \right| = \inf_{m \in \mathcal{M}} \frac{\sigma(m)}{E(m)}.$$

This equality remains valid in infinite-dimensional Hilbert spaces, although the supremum or infimum need not be attained. It says that the largest feasible Sharpe ratio is exactly the smallest coefficient of variation among all valid SDFs.

The economic content is immediate. Since  $\sigma(m)/E(m) = R^f \sigma(m)$ , a large observed Sharpe ratio forces every valid SDF to be sufficiently volatile relative to its mean. Historically, the U.S. equity market has delivered a Sharpe ratio of roughly 0.5 per year, so any admissible SDF must satisfy  $\sigma(m)/E(m) \geq 0.5$ . Standard consumption-based models with power utility and observed aggregate consumption growth typically generate SDFs that are too smooth to meet this restriction unless risk aversion is implausibly high. This is the **equity premium puzzle** of Mehra and Prescott (1985), and the Hansen-Jagannathan bound gives its sharpest quantitative expression.

## The SDF and the Mean-Variance Frontier

The payoff  $x^*$  is traded, and its price is  $p(x^*) = E(x^* x^*) = E(x^{*2}) > 0$  whenever  $x^* \neq 0$ . Its gross return is therefore

$$R^* = \frac{x^*}{E(x^{*2})}.$$

Hansen and Richard (1987) show that  $R^*$  is the traded return with the smallest second moment. Since  $x^*$  prices every traded return at one, for any traded return  $R$ ,

$$E(R^*(R - R^*)) = \frac{E(x^* R) - E(x^* R^*)}{E(x^{*2})} = \frac{1 - 1}{E(x^{*2})} = 0,$$

and hence

$$E(R^2) = E(R^{*2}) + E((R - R^*)^2) \geq E(R^{*2}).$$

Because  $E(R^2) = \sigma^2(R) + E(R)^2$ , every traded return with the same mean as  $R^*$  has at least its variance, so  $R^*$  is a frontier return in the sense of the [portfolio frontier notebooks](#).

Beta pricing with respect to  $R^*$  then follows from the law of one price alone. Since  $x^* = E(x^{*2}) R^*$  prices every traded return  $R^i$  at one,  $E(R^* R^i) = 1/E(x^{*2}) = E(R^{*2})$ . Writing both sides with means and covariances,

$$E(R^*) E(R^i) + \text{Cov}(R^*, R^i) = E(R^*)^2 + \sigma^2(R^*),$$

and with  $\beta_i = \text{Cov}(R^i, R^*)/\sigma^2(R^*)$  this rearranges to

$$E(R^i) = \gamma + \beta_i (E(R^*) - \gamma), \quad \gamma = \frac{E(R^{*2})}{E(R^*)}, \quad (8)$$

provided  $E(R^*) \neq 0$ . This is the [beta-pricing formula](#) in gross returns, with the frontier return  $R^*$  and zero-beta rate  $\gamma$ . When a risk-free asset is traded,  $E(x^*) = p(1) = 1/R^f$ , so  $\gamma = 1/(E(x^{*2}) E(R^*)) = 1/E(x^*) = R^f$ .

The converse is the footnote in the beta-pricing notebook: beta pricing with respect to a frontier return  $R_p$  means that an SDF linear in  $R_p$  prices every traded asset. The mean-variance frontier and the SDF are therefore two descriptions of the law of one price. The SDF in  $X$  is a scaled frontier return, and every frontier return yields an SDF linear in it.

Nothing in this argument makes  $x^*$  positive. In Example 3 it happens to be,  $x^* = [\frac{9}{11}, \frac{13}{11}, \frac{4}{11}, \frac{8}{11}]'$ , but the law of one price does not require it. The next notebook gives an example where  $x^*$  is negative in some state, and shows what the additional assumption of no-arbitrage buys.

## References

Breedon, Douglas T. 1979. "An Intertemporal Asset Pricing Model with Stochastic Consumption and Investment Opportunities." *Journal of Financial Economics* 7 (3): 265–96.

Cochrane, John. 2009. *Asset Pricing: Revised Edition*. Princeton university press.

Hansen, Lars Peter, and Ravi Jagannathan. 1991. "Implications of Security Market Data for Models of Dynamic Economies." *Journal of Political Economy* 99 (2): 225–62.

Hansen, Lars Peter, and Ravi Jagannathan. 1997. "Assessing Specification Errors in Stochastic Discount Factor Models." *Journal of Finance* 52 (2): 557–90.

Hansen, Lars Peter, and Scott F Richard. 1987. "The Role of Conditioning Information in Deducing Testable Restrictions Implied by Dynamic Asset Pricing Models." *Econometrica* 55 (3): 587–613.

Lucas, Robert E. 1978. "Asset Prices in an Exchange Economy." *Econometrica* 46 (6): 1429–45.

Mehra, Rajnish, and Edward C. Prescott. 1985. "The Equity Premium: A Puzzle." *Journal of Monetary Economics* 15 (2): 145–61.